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Networks
DIY (Do-It-Yourself) Math Workshop

Dr. Ralph deLaubenfels

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Teacher-Scholar Institute

Columbus, Ohio

2020

DIY (Do-It YOURSELF)

NETWORKS WORKSHOP

As with all DIY Workshops,

Writing / drawings in red ——
are written on a chalkboard
& possibly spoken;

Writing in quotes in black
"——" is said out loud to
students, not written;

Writing not in quotes in black
—— is suggested, not spoken
or written

TABLE OF CONTENTS

I. PREREQUISITES

Counting lines

Knowing the definitions of whole numbers $\{0, 1, 2, 3, \ldots\}$, even numbers $\{0, 2, 4, 6, 8, \ldots\}$, & odd numbers $\{1, 3, 5, 7, \ldots\}$

II. MATERIALS NEEDED

Two or more chalkboards, that we will call Board 1, Board 2, etc.

For each participant, print out Worksheets 1-4 at the end of this exposition. Also print out one or more copies of the last page of this exposition & cut each copy into 8 small cards as on the next 2 pages.

Number of Odd Vertices	Euler Path?	Euler Circuit?	Number of Odd Vertices	Euler Path?	Euler Circuit?
0			0		
2			2		
More Than 2			More Than 2		

Number of Odd Vertices	Euler Path?	Euler Circuit?	Number of Odd Vertices	Euler Path?	Euler Circuit?
0			0		
2			2		
More Than 2			More Than 2		

Number of Odd Vertices	Euler Path?	Euler Circuit?	Number of Odd Vertices	Euler Path?	Euler Circuit?
0			0		
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Number of Odd Vertices	Euler Path?	Euler Circuit?	Number of Odd Vertices	Euler Path?	Euler Circuit?
0			0		
2			2		
More Than 2			More Than 2		

cut along bolder lines

Number of Odd Vertices	Euler Path?	Euler Circuit?
0		
2		
More Than 2		

Number of Odd Vertices	Euler Path?	Euler Circuit?
0		
2		
More Than 2		

Number of Odd Vertices	Euler Path?	Euler Circuit?
0		
2		
More Than 2		

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0		
2		
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0		
2		
More Than 2		

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0		
2		
More Than 2		

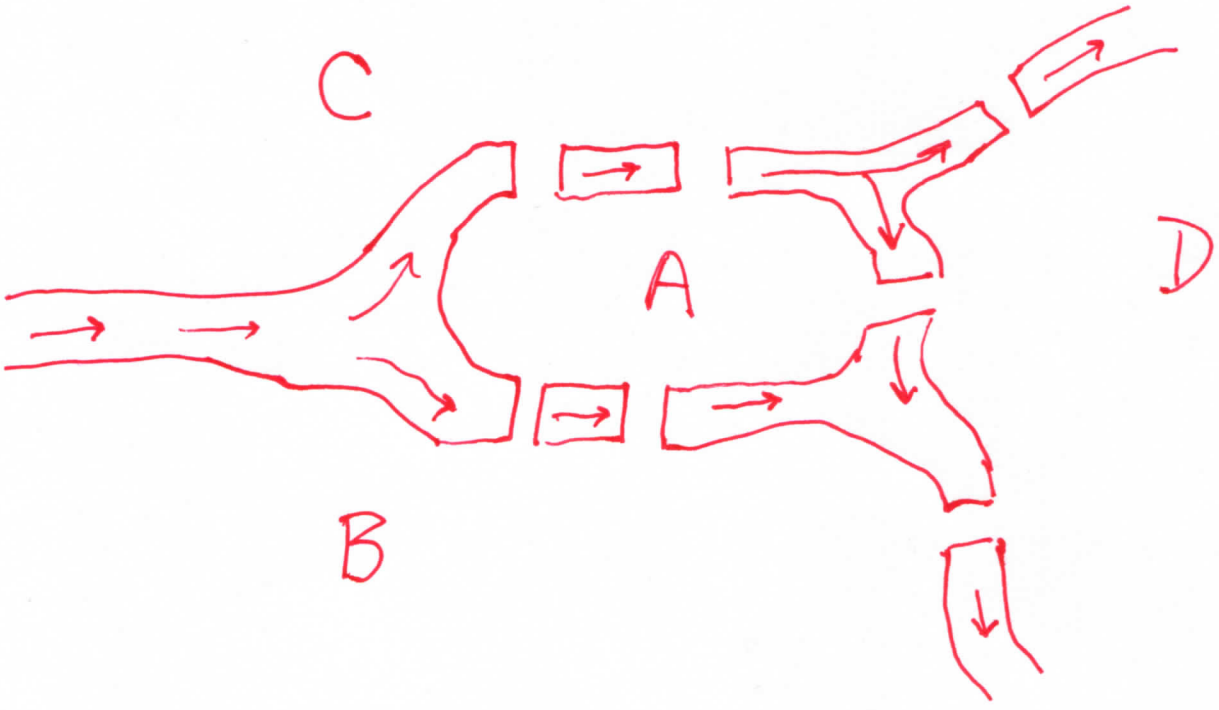
Number of Odd Vertices	Euler Path?	Euler Circuit?
0		
2		
More Than 2		

Number of Odd Vertices	Euler Path?	Euler Circuit?
0		
2		
More Than 2		

III. BRIDGES of KÖNIGSBERG

Board 1

BRIDGES of KÖNIGSBERG



(arrows are rivers, A, B, C, & D are land masses)

Board 2

GOAL:

Create a walking tour that goes over each bridge exactly once

("Avoid boredom of walking over the same bridge twice")

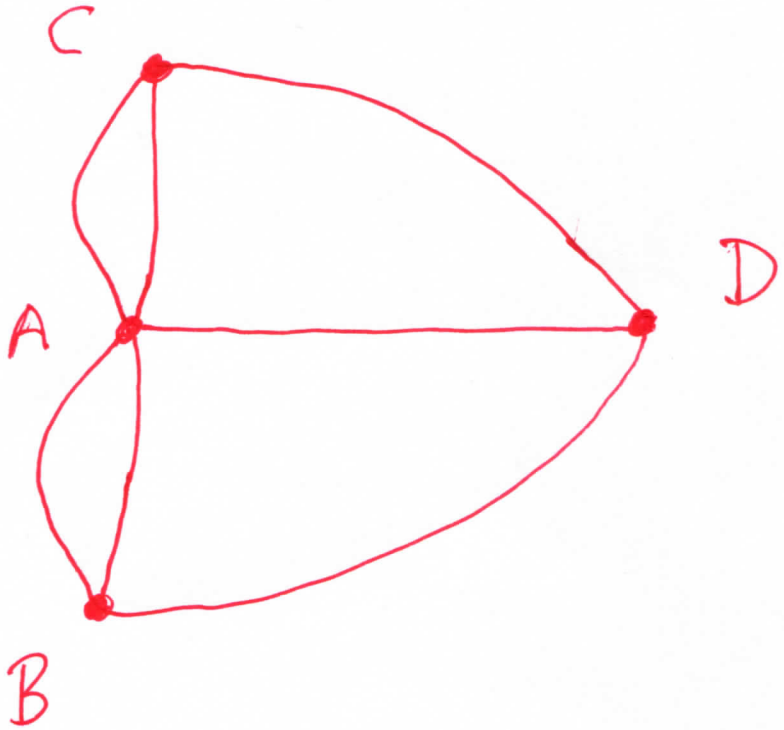
Trace some paths with finger, on Board 1, attempting to cross all bridges.

"Let's simplify our picture of the Königsberg bridges"

new Board 2

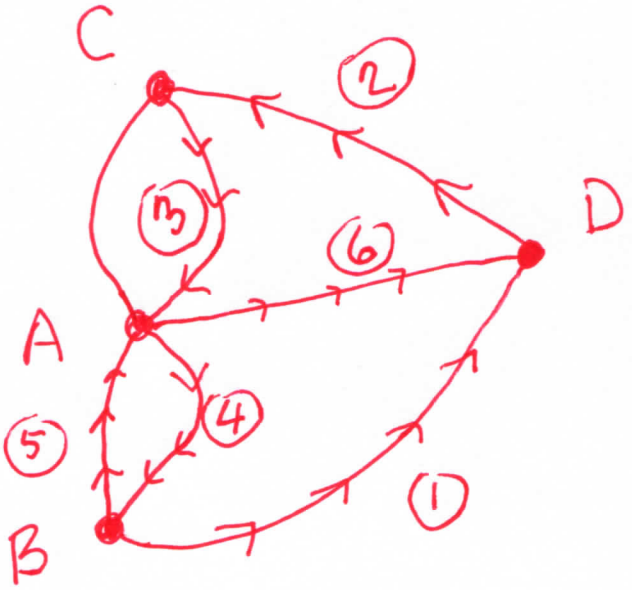
• for land mass

—— for bridge



"We can draw some walking tours"

new Board 1



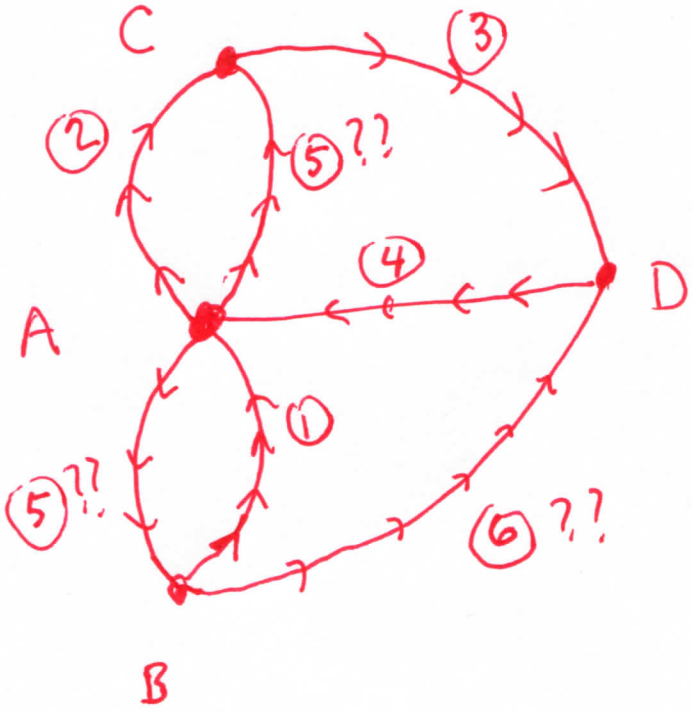
BDCABADB

(B to D to C to A to B to A to D)

Trace path with finger;

"looks like one bridge isn't travelled" (point to untravelled bridge)

"Try again." (Trace with finger)



"Again, looks like at least one bridge is untravelled."

"Guessing looks like hard work. How can we know if we're wasting our time, looking for a path over all bridges when there is no such path?"

IV. NETWORKS

new Board 1

dot • is vertex

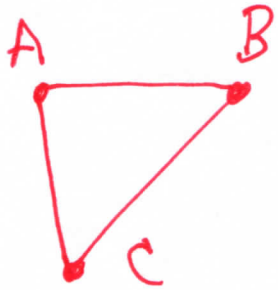
line —— is edge

Network or graph is vertices* connected by edges.

*"vertices" is plural of vertex

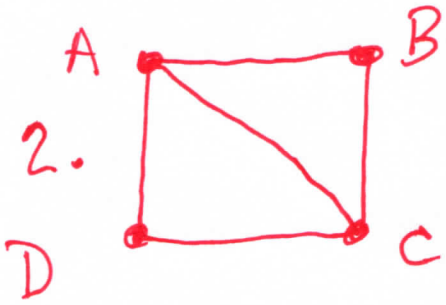
new Board 2

Examples 1.



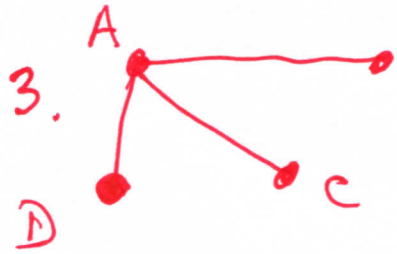
(3 vertices, 3 edges)

2.



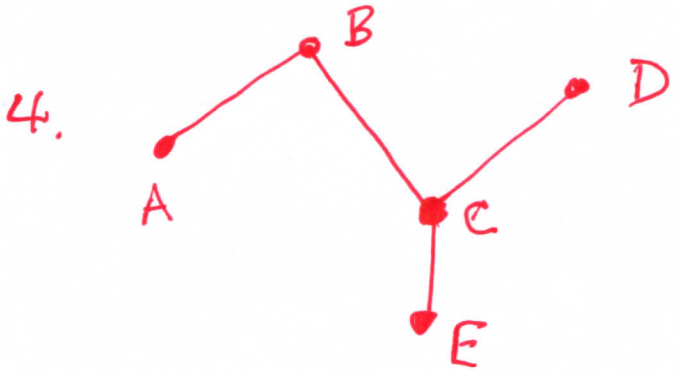
(4 vertices, 5 edges)

3.



(4 vertices, 3 edges)

4.



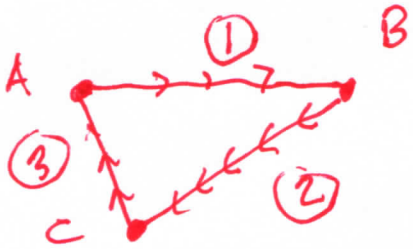
(5 vertices, 4 edges)

V. EULER PATHS

new Board 1

Euler Path: travel all edges without jumping or retracing.

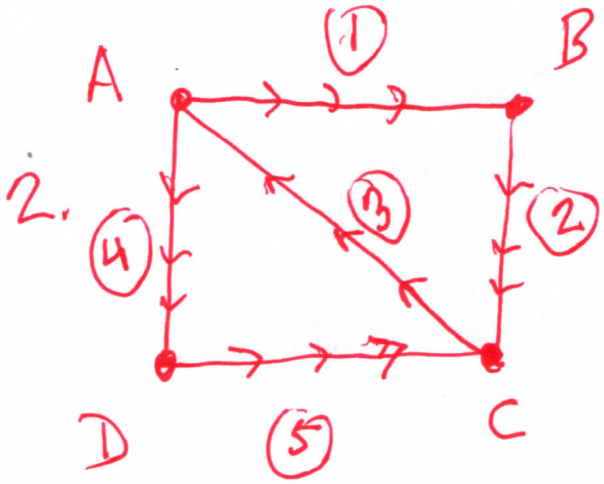
Examples (from other board)

1. 

trace path with finger.

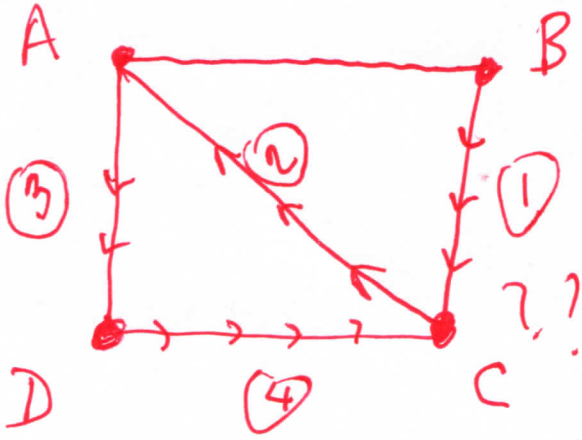
ALSO WRITTEN: ABCA, short for "from A to B, from B to C, from C to A"

2.



trace with finger

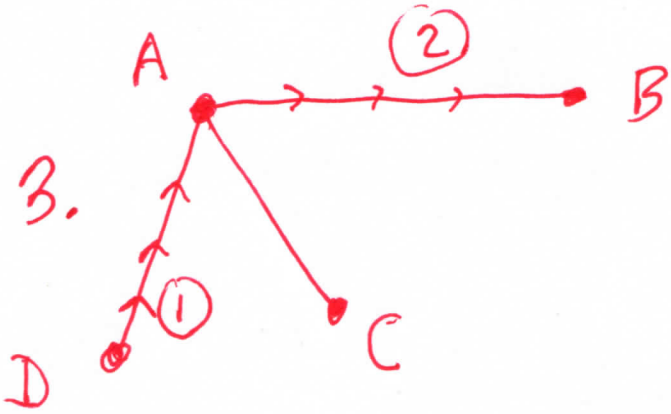
"Start path at B?"



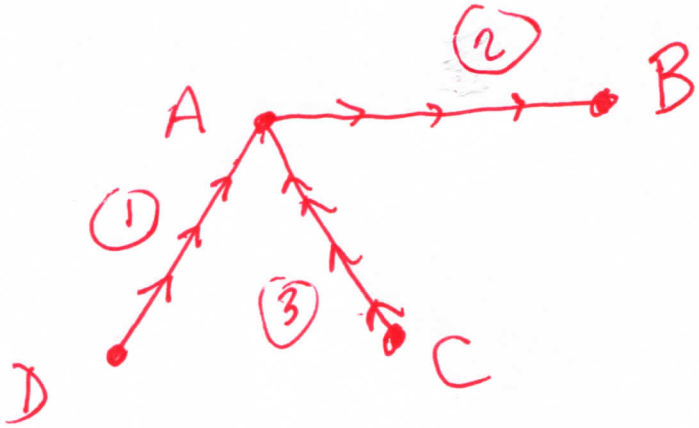
trace with finger

?! WHERE TO GO, after (4)?

"It can be shown (soon) that in Example 2 it makes a difference where you start, while in Example 1 it does not."

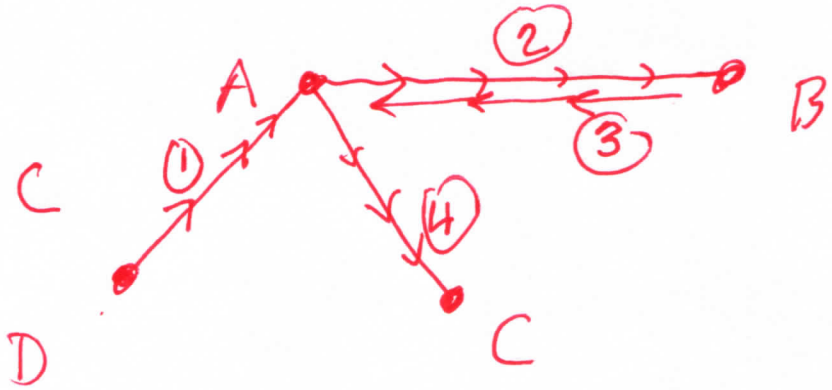


edge missed



Not legal; we jumped from B to C

DABAC



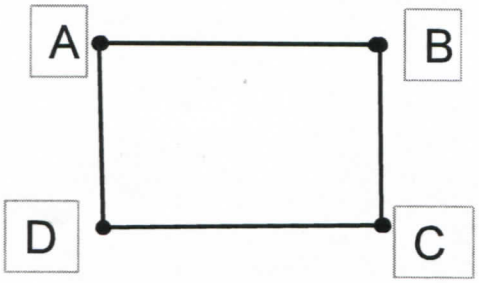
Also not legal; edge from A to B got retraced

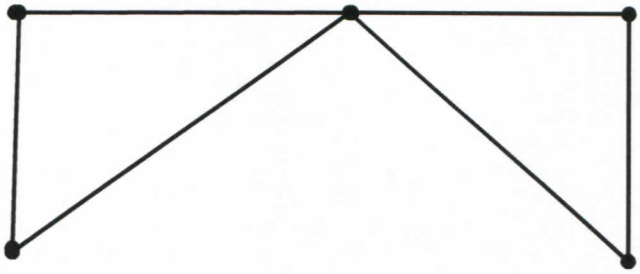
CAN SHOW (later):

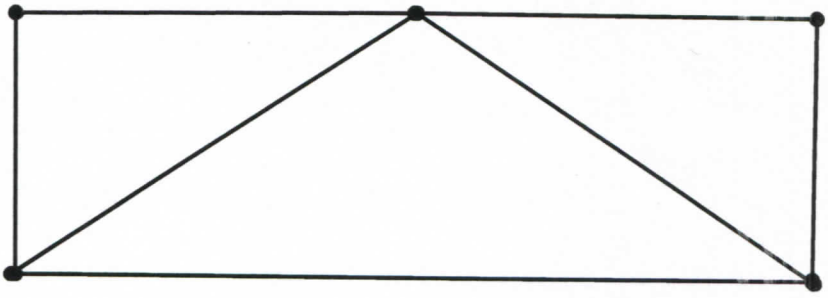
Examples 3 + 4 have no Euler paths.

HAND OUT copy of Worksheet 1 to each participant; let each participant (student) look for Euler paths, for a while

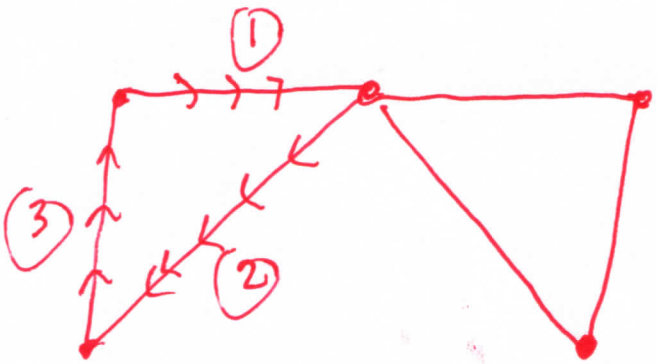
Networks
Worksheet 1





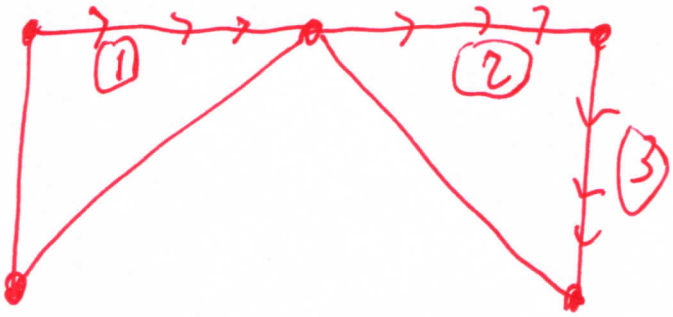


new Board 2



What to do after (3)?

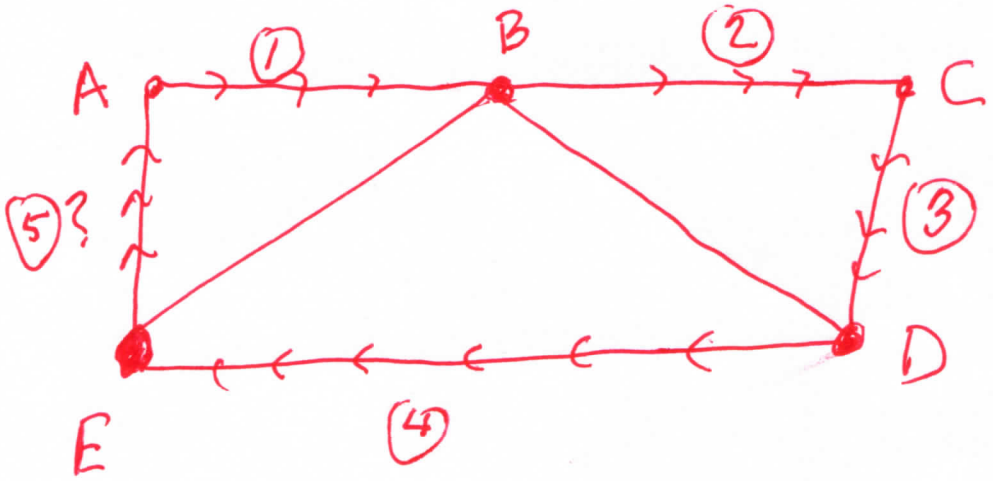
RECOMMEND circling around outside, then spiralling in; e.g.



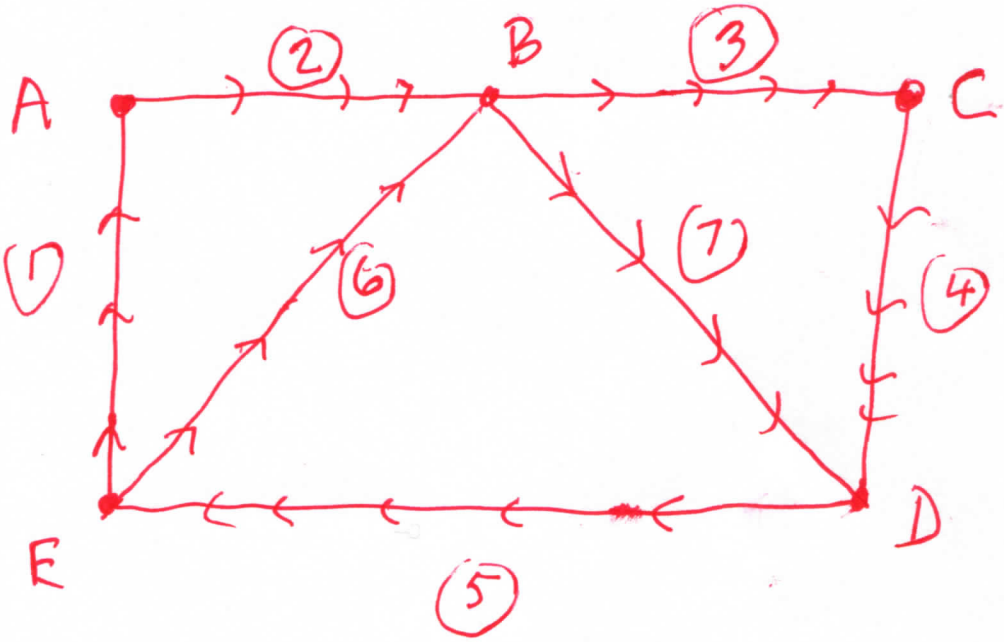
etc.

new Board 1

last example:



NEED to start at D or E; e.g.

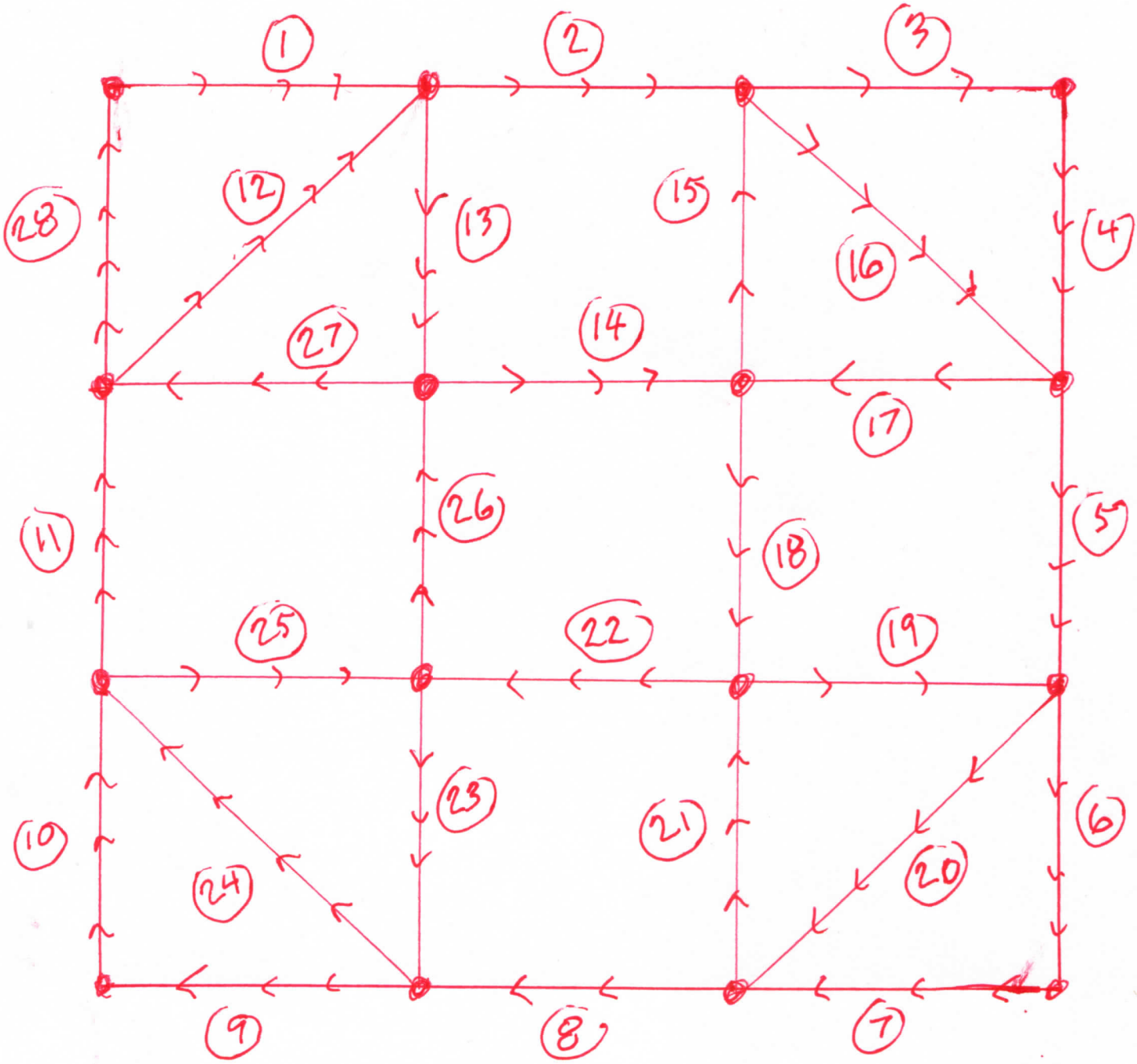


"Let's get an Euler path for a larger network."

Do this on a board with extensive student involvement & suggestions.

(After drawing network)

"We'll start at the upper left & spiral inside slowly"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28

HAND OUT copy of Worksheet 2 to each participant.

Have students work in groups on boards, looking for Euler paths; you may stroll around & drop hints.

Eventually mention that, in last two networks, it makes a difference where you start.

Eventually make sure all groups have correct Euler paths.

Networks
Worksheet 2

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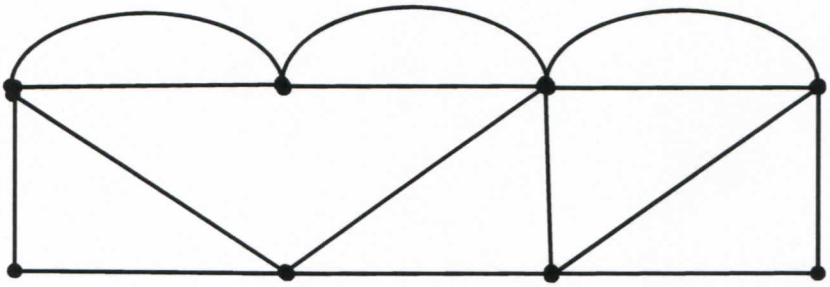

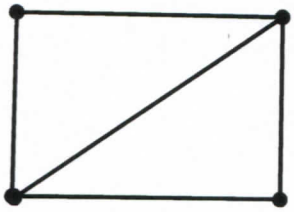

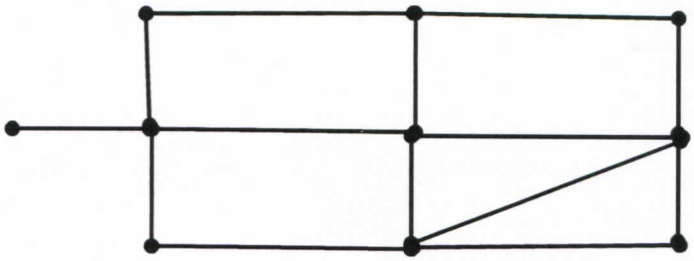

Networks
Worksheet 2

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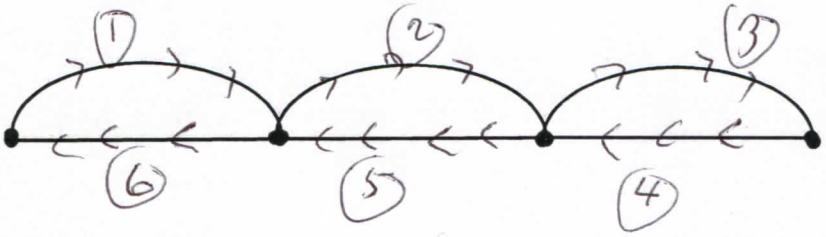
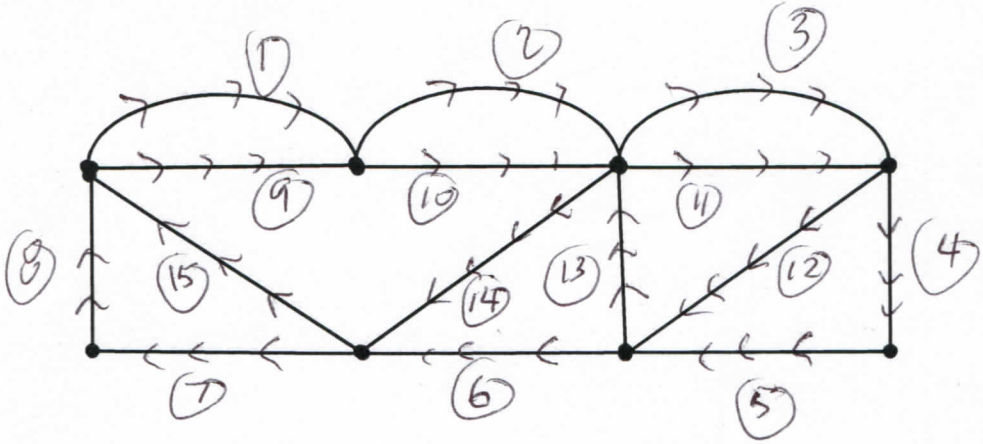
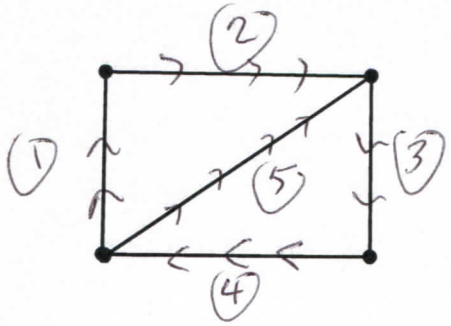
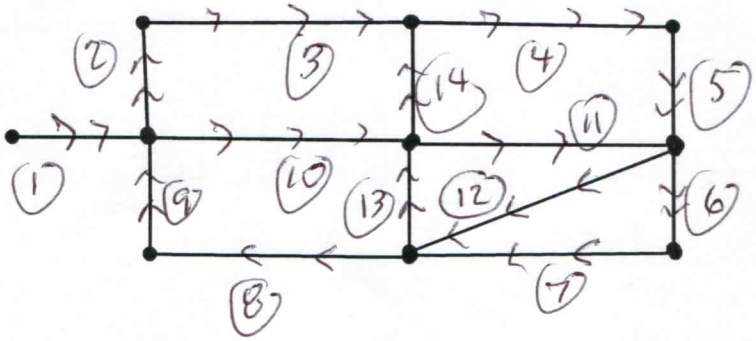
Euler paths drawn; there are many other correct answers









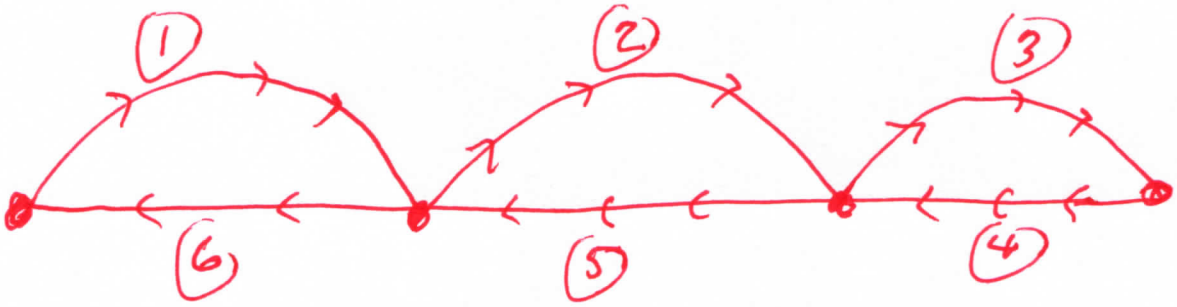
"Recall walking tour of Königsberg bridges. If you drove & parked your car at the first bridge, you'd like your tour to end at the first bridge, so you can drive home."

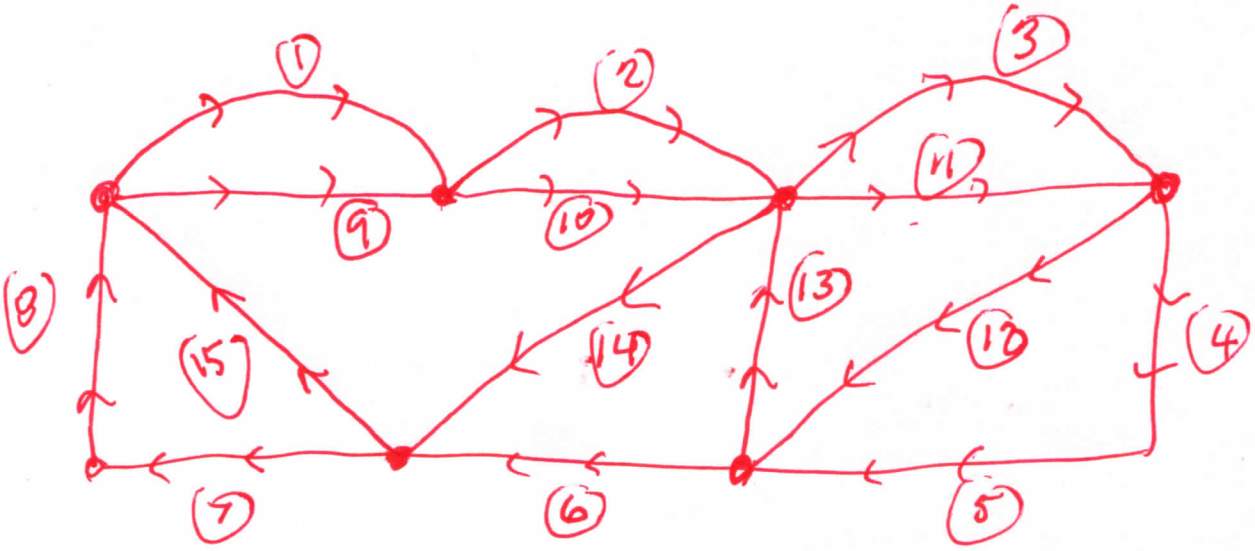
new Board 2

An Euler circuit is an Euler path that ends at the same vertex where it began.

"In Worksheet 2 only the first two networks have Euler circuits."

new Board 1





VI HOW TO TELL if EULER PATH or CIRCUIT EXISTS

"How can we tell when a network has an Euler path or circuit? Just because we've failed to find one doesn't mean it doesn't exist; perhaps we just had bad luck. On the other hand, we'd hate to spend a long time looking for something that doesn't exist."

"For a clue, let's focus on three examples."

new Board 1

Examples (A) 

has Euler circuit, hence Euler path (trace with finger)

(B) 

has Euler path, no Euler circuit

(C) 

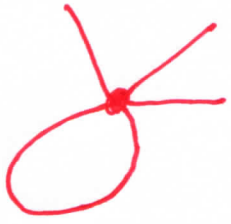
no Euler path, hence no Euler circuit

"We'll come back to these after a few more definitions."

"What turns out to be important is the degree of a vertex."

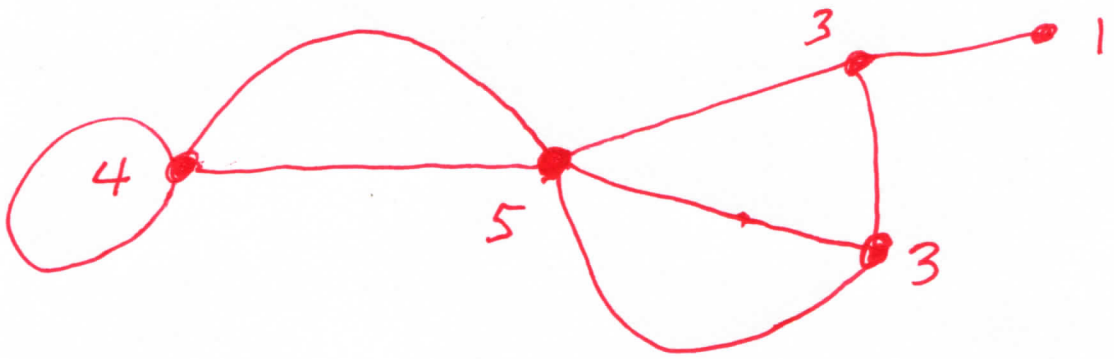
new Board 2

The degree of a vertex is the number of edges emanating from it.

Example  vertex has degree 5.

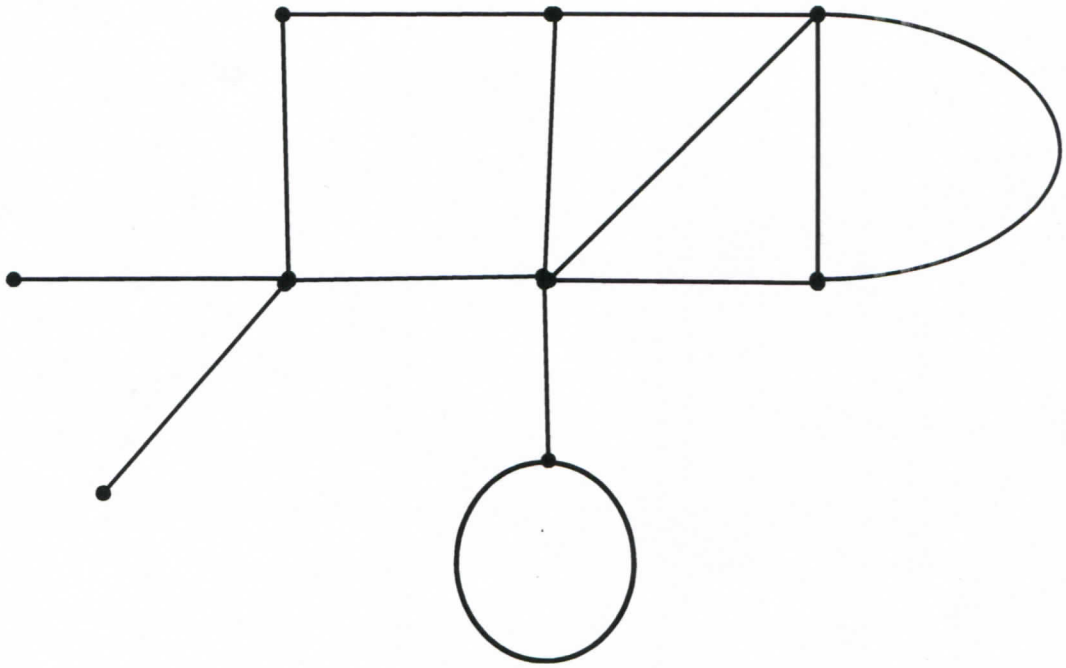
new Board 1

Example (numbers are degrees)



HAND OUT Worksheet 3 (get degrees of vertices); after students have worked on it for a while, put correct answers on a Board.

Networks
Worksheet 3



Networks
Worksheet 3

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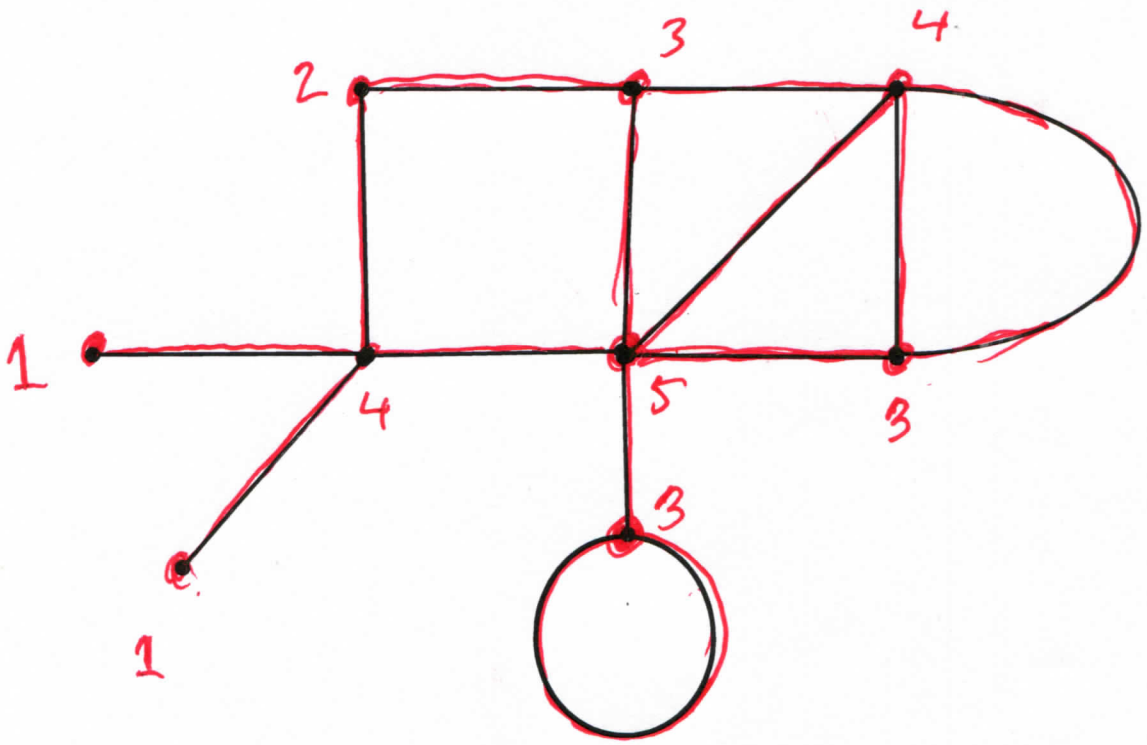
ANSWERS



(new Board 2)

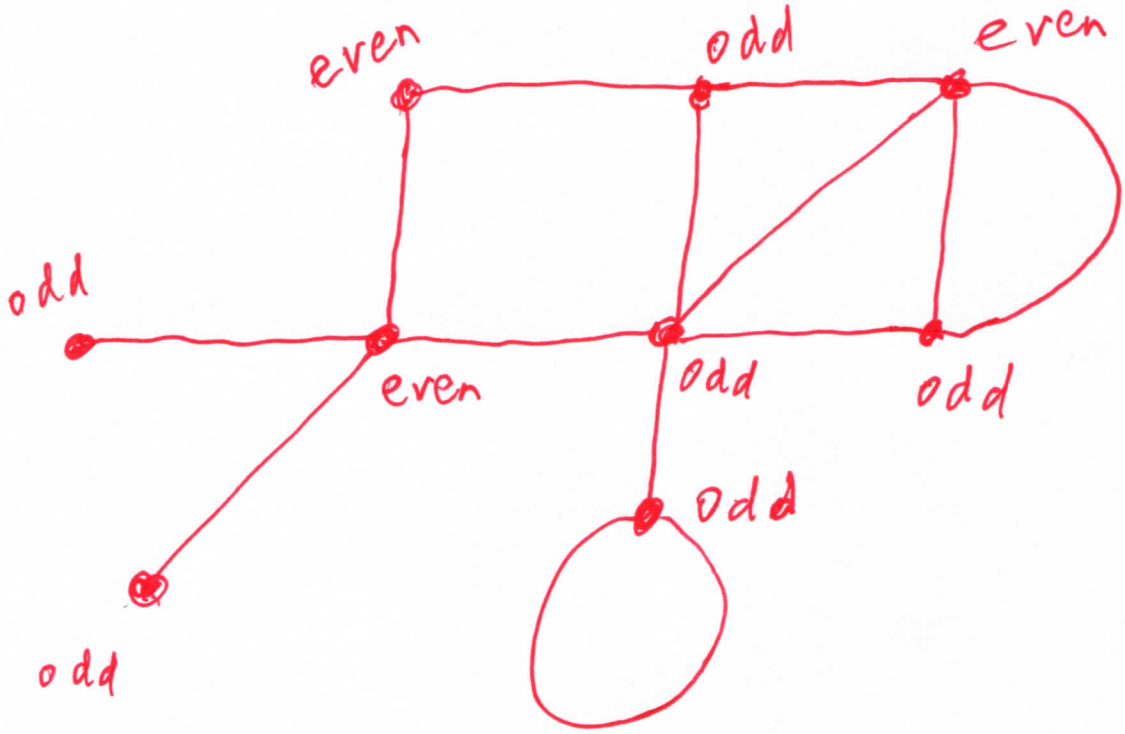
new Board 1

Even vertex is a vertex whose degree is even (0, 2, 4, 6, ...)

Odd vertex is a vertex whose degree is odd.

"On Worksheet 3 (Board 2), label odd & even vertices; how many even? how many odd?"

new Board 2

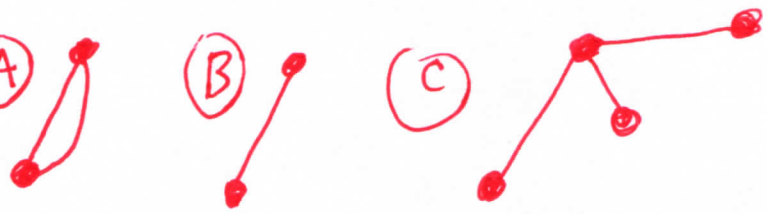


3 even vertices

6 odd vertices

new Board 2

Recall (A) (B) (C)



Network	# of odd vertices	Euler path?	Euler circuit?
(A)	0	YES	YES
(B)	2	YES	NO
(C)	more than 2	NO	NO

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"The last three columns are always true"

HAND OUT card to each student

Number of Odd Vertices	Euler Path?	Euler Circuit?
0		
2		
More Than 2		

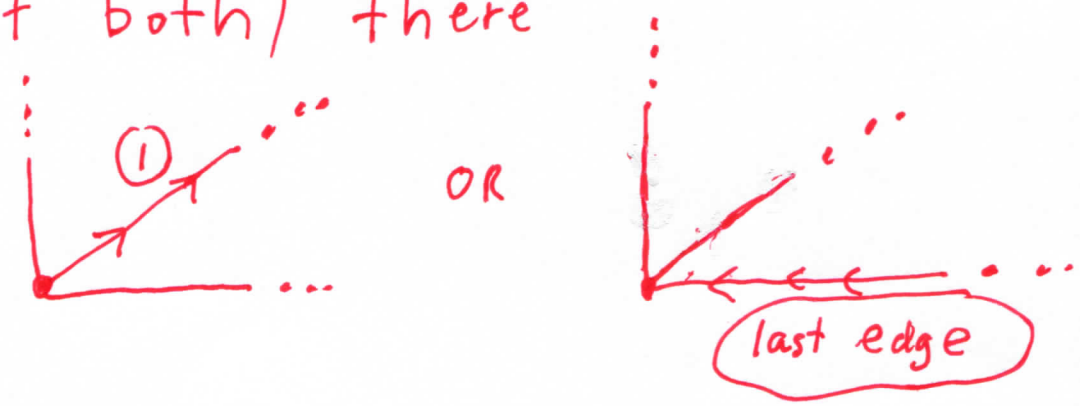
Students fill in blanks:

Number of Odd Vertices	Euler Path?	Euler Circuit?
0	YES	YES
2	YES	NO
More Than 2	NO	NO

new Board 1

UNDERLYING FACT:

If a vertex is odd, an Euler path must begin or end (but not both) there



IF ASKED: "The number of odd vertices in a network is even, since the sum of the degrees of all vertices is even."

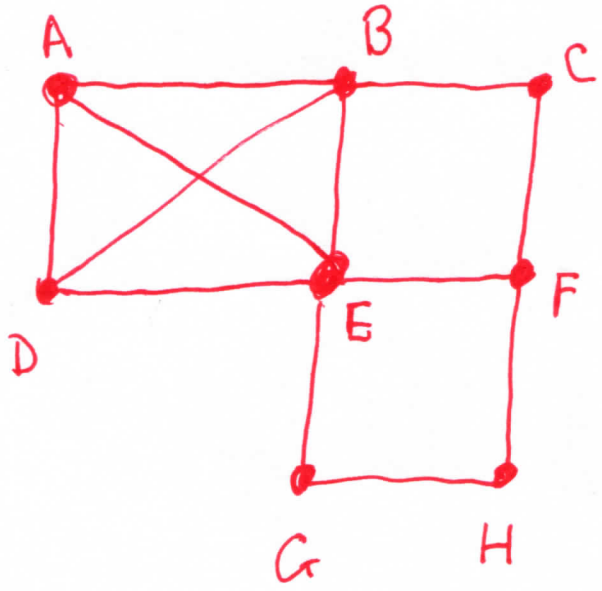
new Board 2

Example: Is there an Euler path? If so, where must it begin & end? Is there an Euler circuit?

STRATEGY: Identify odd vertices; use card & "Underlying Fact," & fact that Euler circuit can begin anywhere.

new Board 1

1.

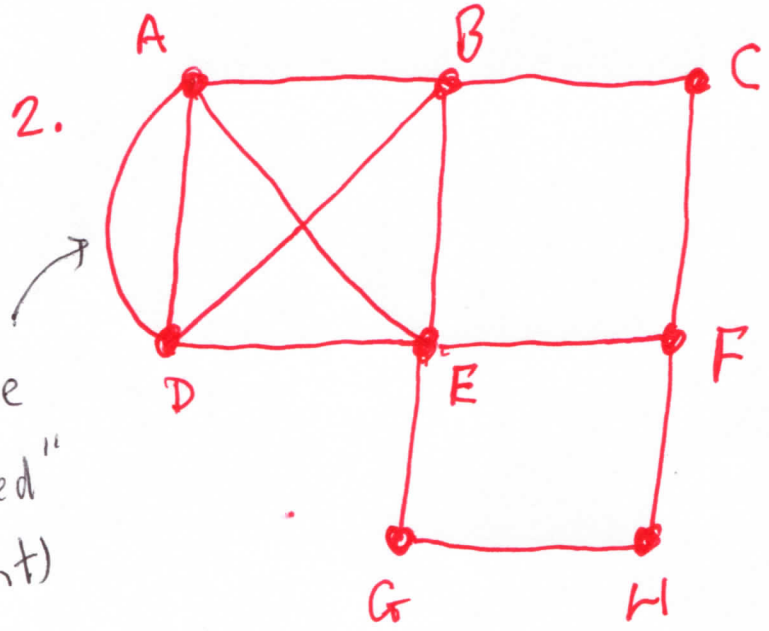


let students work; then identify (still on Board 1)

"odd vertices are A, D, E, & F, more than 2 odd vertices, so no Euler path or circuit"

new Board 1

2.



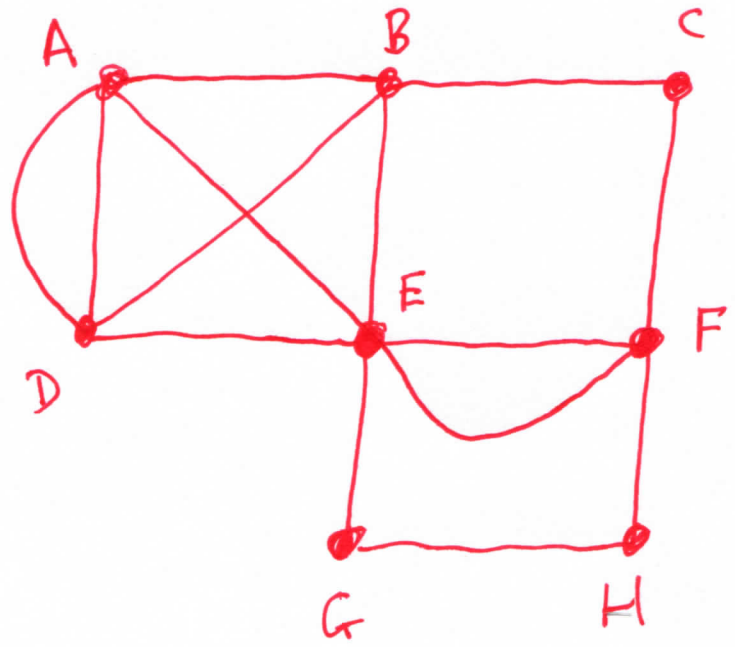
"edge added" (point)

let students work, then identify

"odd vertices are E & F, 2 odd vertices, so no Euler circuit, but there is an Euler path, which must begin at E or F & end at E or F, different from beginning vertex."

new Board 1

3.

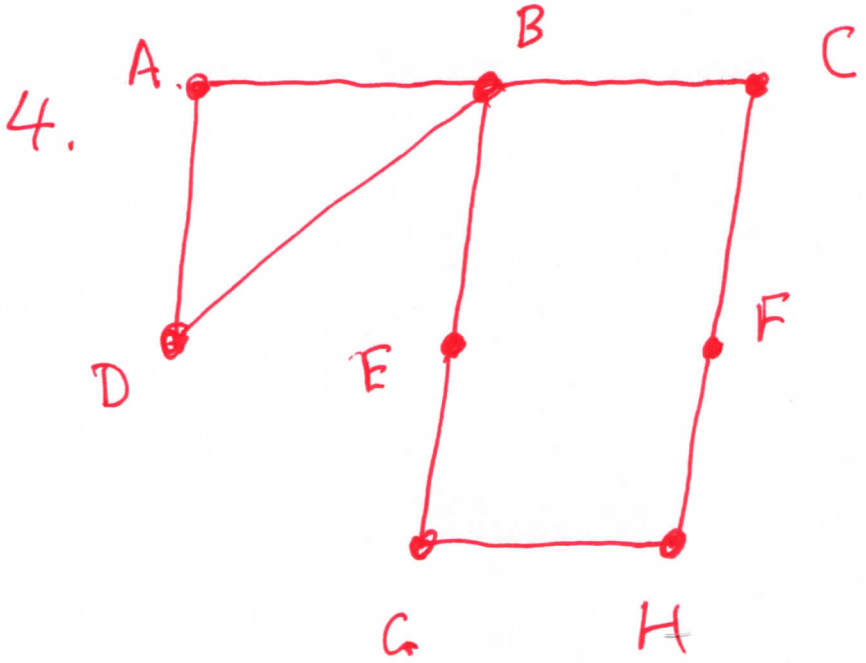


"another edge added" (point)

let students work; then assert "no odd vertices, so Euler circuit, hence Euler path, exists, & may begin anywhere."

new Board 1

4.

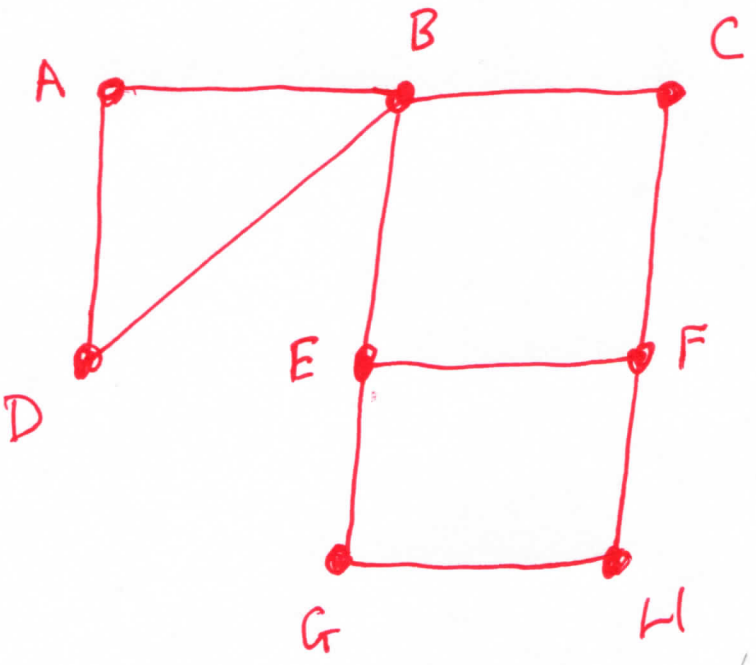


"some edges removed" (point)

let students work; then assert "no odd vertices, so Euler circuit, hence Euler path, exists, & may begin anywhere."

new Board 1

5.



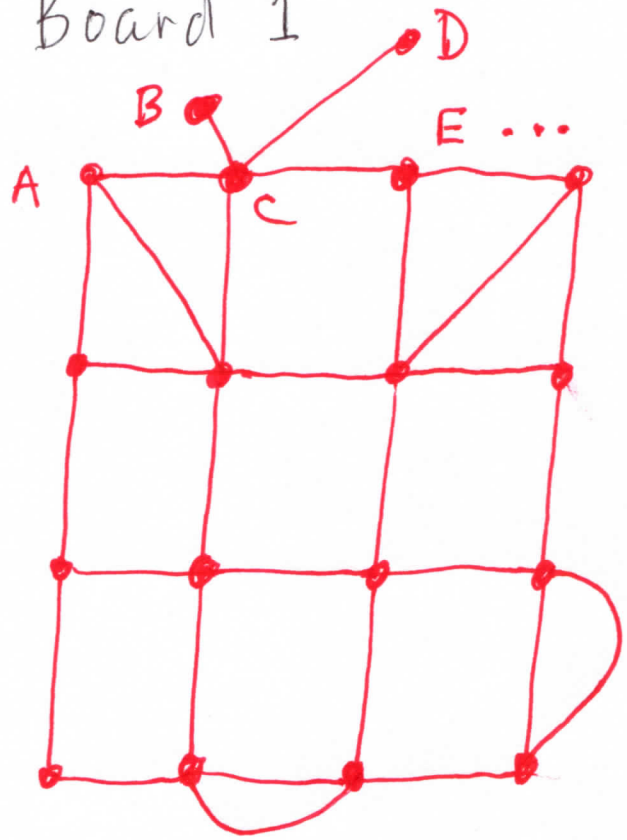
"edge added to Ex. 4; OR, some edges removed from Ex. 1"

let students work; then identify

"odd vertices are E & F, 2 odd vertices, so no Euler circuit, but there is an Euler path, which must begin at E or F & end at E or F, different from beginning vertex."

new Board 1

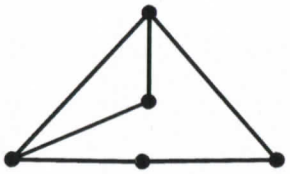
6.



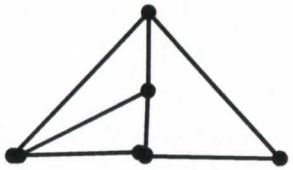
"Vertices A, B & D are odd, so we have more than two odd vertices; no Euler path or Euler circuit."

HAND OUT Worksheet 4, let students work on it.

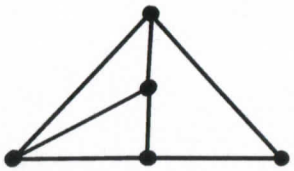
Networks
Worksheet 4



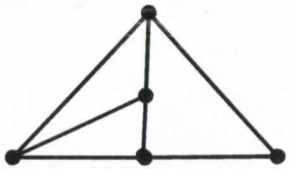
Euler Path? Yes or No
Euler Circuit? Yes or No



Euler Path? Yes or No
Euler Circuit? Yes or No



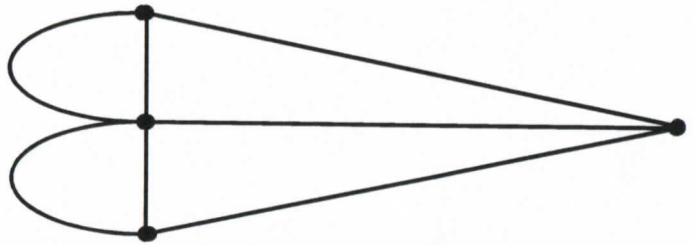
Add an edge to make an Euler Path.



Add two edges to make an Euler Circuit.

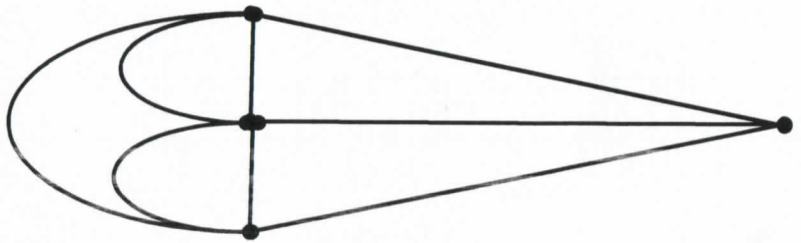
more than one right answer

Königsberg:



Can you find an Euler Path?
Can you find an Euler Circuit?

Königsberg later:

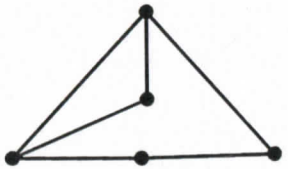


Can you find an Euler Path?
Can you find an Euler Circuit?

Networks
Worksheet 4

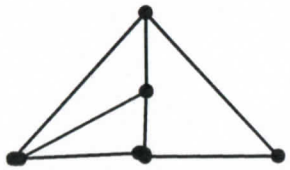
ANSWERS

2 odd vertices



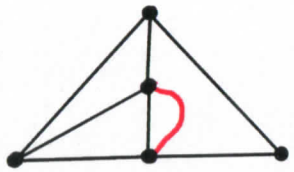
Euler Path? Yes or No
Euler Circuit? Yes or No

4 odd vertices



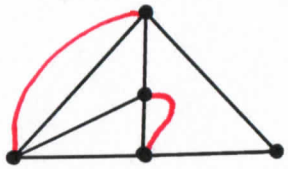
Euler Path? Yes or No
Euler Circuit? Yes or No

2 odd vertices



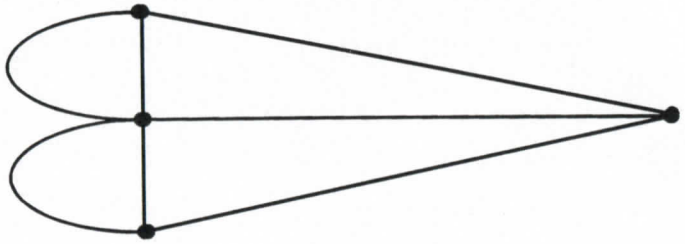
Add an edge to make an Euler Path.

no odd vertices



Add two edges to make an Euler Circuit.

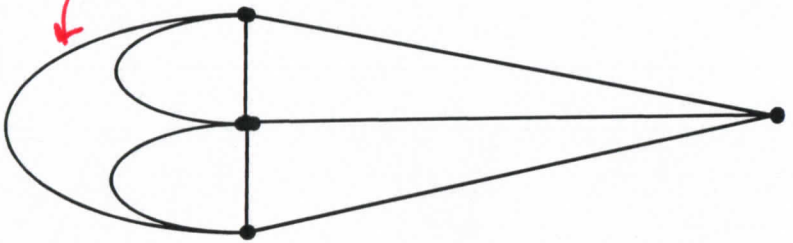
Königsberg:



Can you find an Euler Path? NO (4 odd vertices)
Can you find an Euler Circuit? NO

another bridge

Königsberg later:



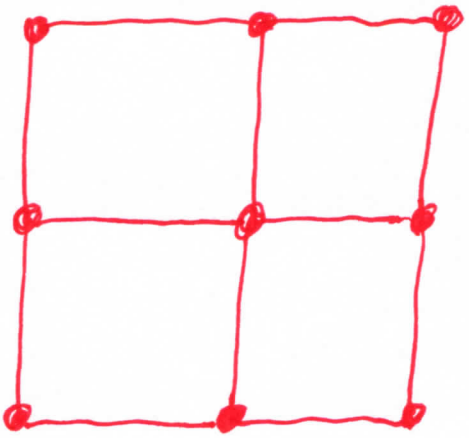
Can you find an Euler Path? YES (2 odd vertices)
Can you find an Euler Circuit? No

p. 45

One at a time, draw network on board, then have a student come up & add or subtract edges.

new Board 2

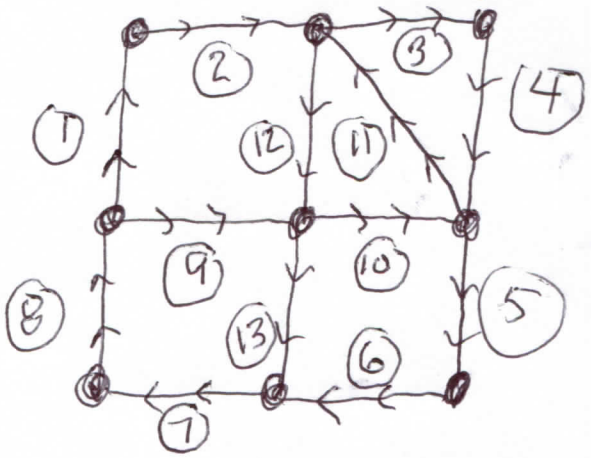
(a)



Add edge, so that Euler path exists.

Then draw an Euler path.

(one answer:

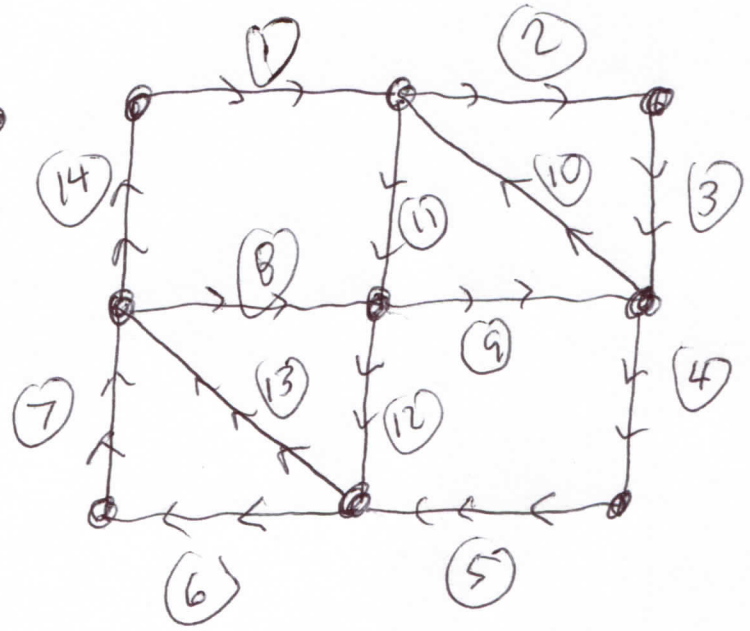


)

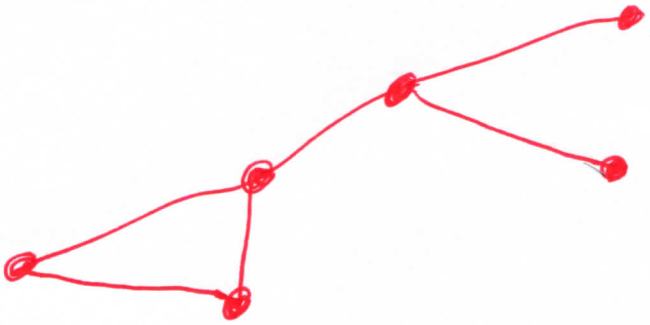
new Board 1

(b) For network at beginning of (a), add two edges, so that Euler circuit exists. Then draw an Euler circuit.

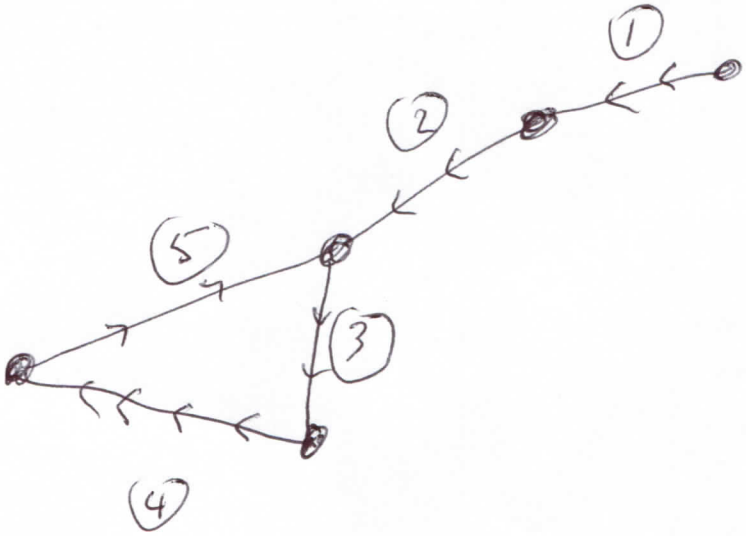
one answer



(c) In the following, subtract an edge & vertex so that an Euler path exists. Then draw an Euler path



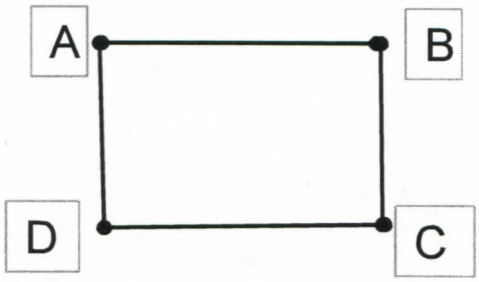
one answer:

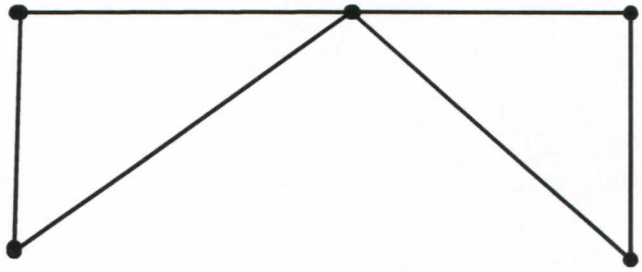


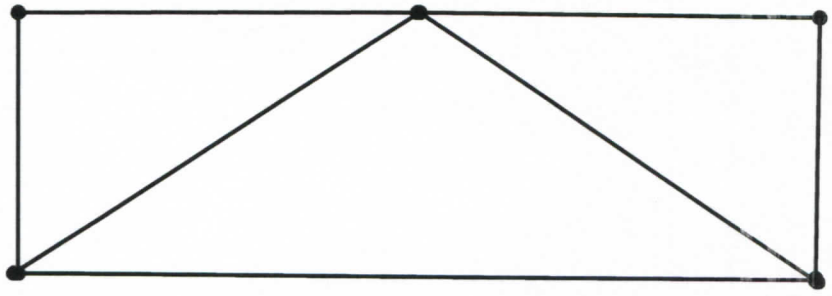
Have pairs of students come to a board. In each pair, one student draws a network & the other student adds a minimal number of edges so that an Euler path exists. Then they should draw an Euler path.

Do the same as above, with "circuit" replacing "path".

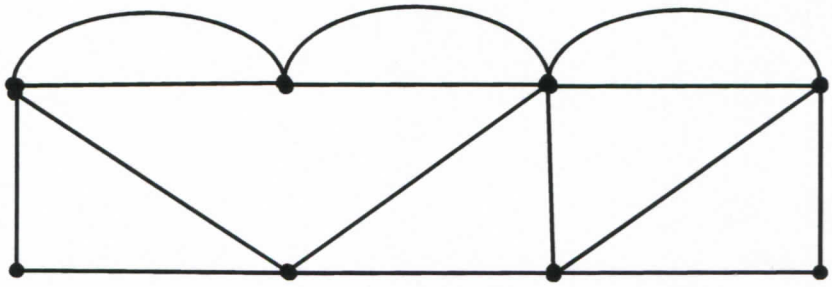
Networks
Worksheet 1

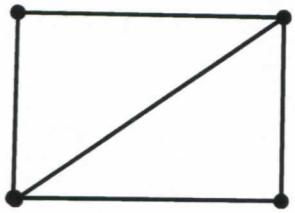


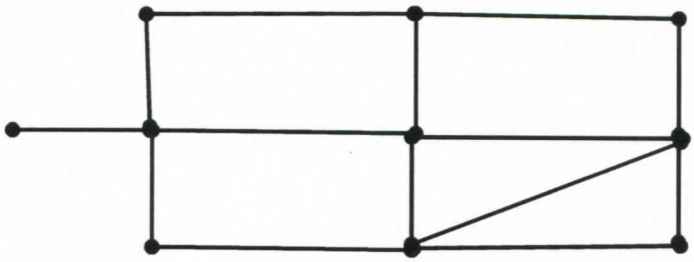




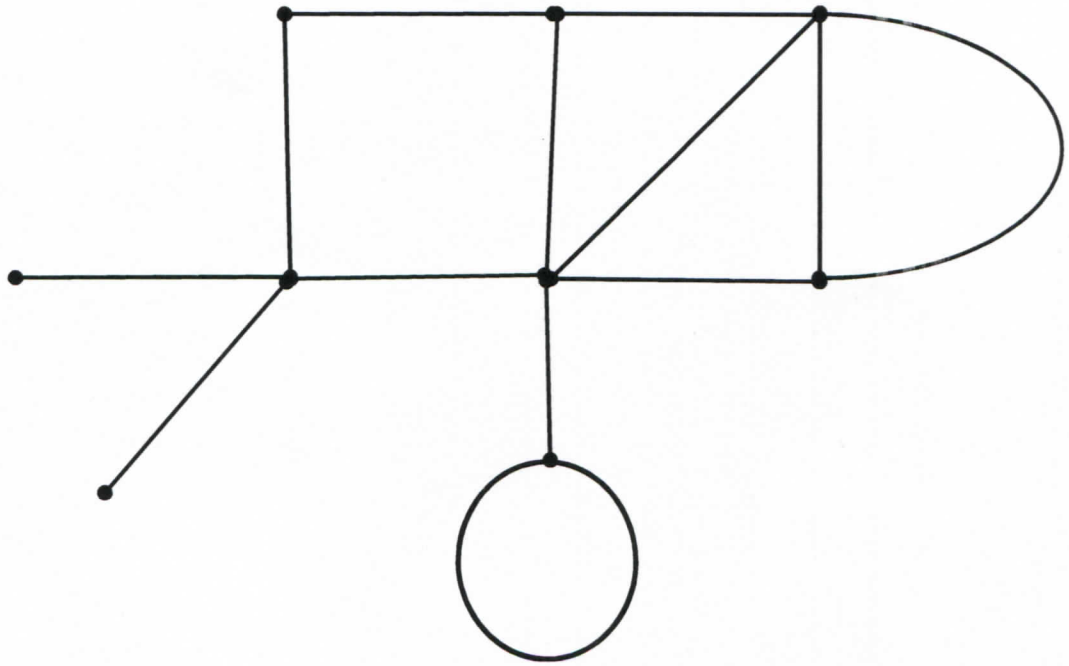
Networks
Worksheet 2



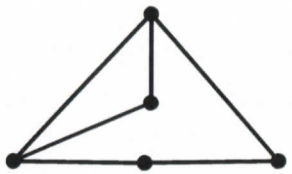




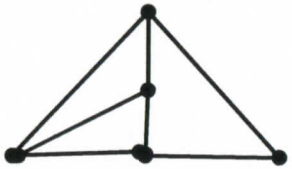
Networks
Worksheet 3



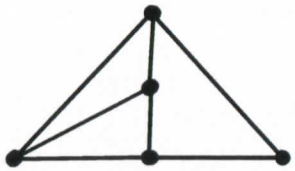
Networks
Worksheet 4



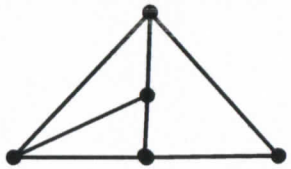
Euler Path? Yes or No
Euler Circuit? Yes or No



Euler Path? Yes or No
Euler Circuit? Yes or No

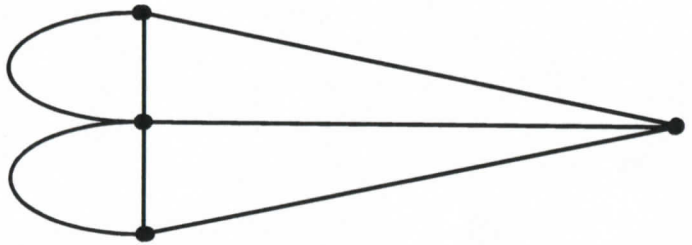


Add an edge to make an Euler Path.



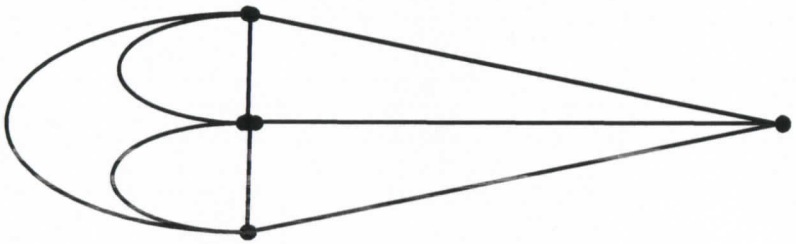
Add two edges to make an Euler Circuit.

Königsberg:



Can you find an Euler Path?
Can you find an Euler Circuit?

Königsberg later:



Can you find an Euler Path?
Can you find an Euler Circuit?

Number of Odd Vertices	Euler Path?	Euler Circuit?
0		
2		
More Than 2		

Number of Odd Vertices	Euler Path?	Euler Circuit?
0		
2		
More Than 2		

Number of Odd Vertices	Euler Path?	Euler Circuit?
0		
2		
More Than 2		

Number of Odd Vertices	Euler Path?	Euler Circuit?
0		
2		
More Than 2		

Number of Odd Vertices	Euler Path?	Euler Circuit?
0		
2		
More Than 2		

Number of Odd Vertices	Euler Path?	Euler Circuit?
0		
2		
More Than 2		

Number of Odd Vertices	Euler Path?	Euler Circuit?
0		
2		
More Than 2		

Number of Odd Vertices	Euler Path?	Euler Circuit?
0		
2		
More Than 2		