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Graphing, Calculus Style, Without Calculus
MATHematics MAGnification™

Dr. Ralph deLaubenfels

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Teacher-Scholar Institute

Columbus, Ohio

2019

GRAPHING, CALCULUS STYLE, without CALCULUS

This is one of a series of very short books on math, statistics, and physics called "Math Magnifications." The "magnification" refers to focusing on a particular topic that is pivotal in or emblematic of mathematics.

OUTLINE

One of the more challenging parts of single-variable calculus is using the first and second derivatives of a function to draw its graph. In this Magnification, we give a simple strategy for graphing, without having to mention derivatives, although the same strategy works in the calculus setting, by assuming we have *distance* as a function of time. We then use minimal information about velocity and acceleration to graph our function.

Many examples are given. We also illustrate how, as in calculus, information about the graph of a function tells us where maxima or minima might occur and gives us long-term behavior.

Prerequisites for this magnification are algebra ([1] is more than sufficient), including the definition of the graph of a function.

Students who have had calculus will recognize velocity and acceleration as first and second derivatives, respectively, whose positivity or negativity will give extensive information about the graph of a function, including a near-complete picture of the *shape* of the graph. Calculus is *not* a prerequisite for this Magnification; however, the techniques introduced in this Magnification apply equally well to graphing in a calculus course.

1. INTRODUCTION

Of interest will be the distance to the sun of a glass spaceship (think of Wonder Woman in outer space) at arbitrary times in the future. Assume that no instruments that could give us a clue about the distance to the sun work; we have only our human senses to use. Specifically, we can tell if our velocity away from the sun is positive or negative, by seeing if the sun appears to be getting smaller (positive velocity) or larger (negative velocity), and we can tell if our acceleration (rate of change of velocity) is positive or negative by feeling the force produced by our acceleration.

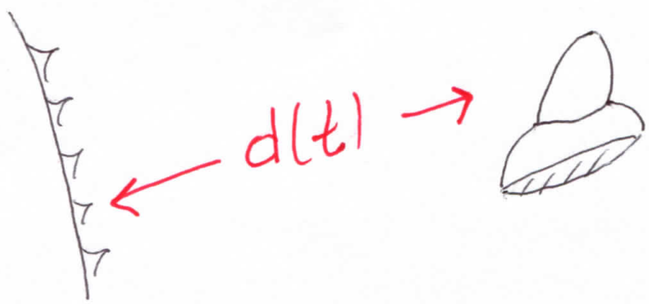
Assume our spaceship is traveling in a straight line to, or away from, the sun.

The information just described will be *all* we know about our distance to the sun. Our goal is to say (or better yet, draw) as much as we can about said distance.

Terminology, Assumptions, and Goals 1.1. Throughout this Magnification, for any nonnegative t,

$d(t)$ is the distance, in meters, our spaceship is from the sun

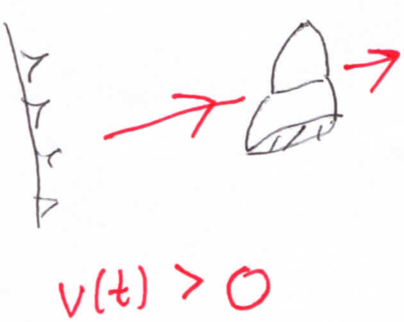
t seconds after leaving the earth.

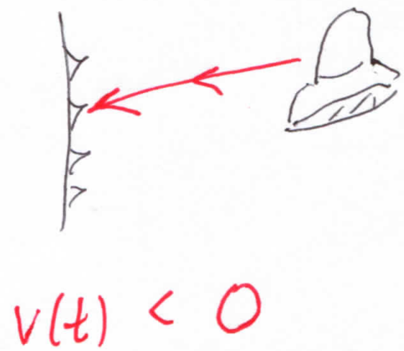


Also denote by

$v(t)$ the velocity of our spaceship away from the sun, in meters per second,

t seconds after leaving the earth





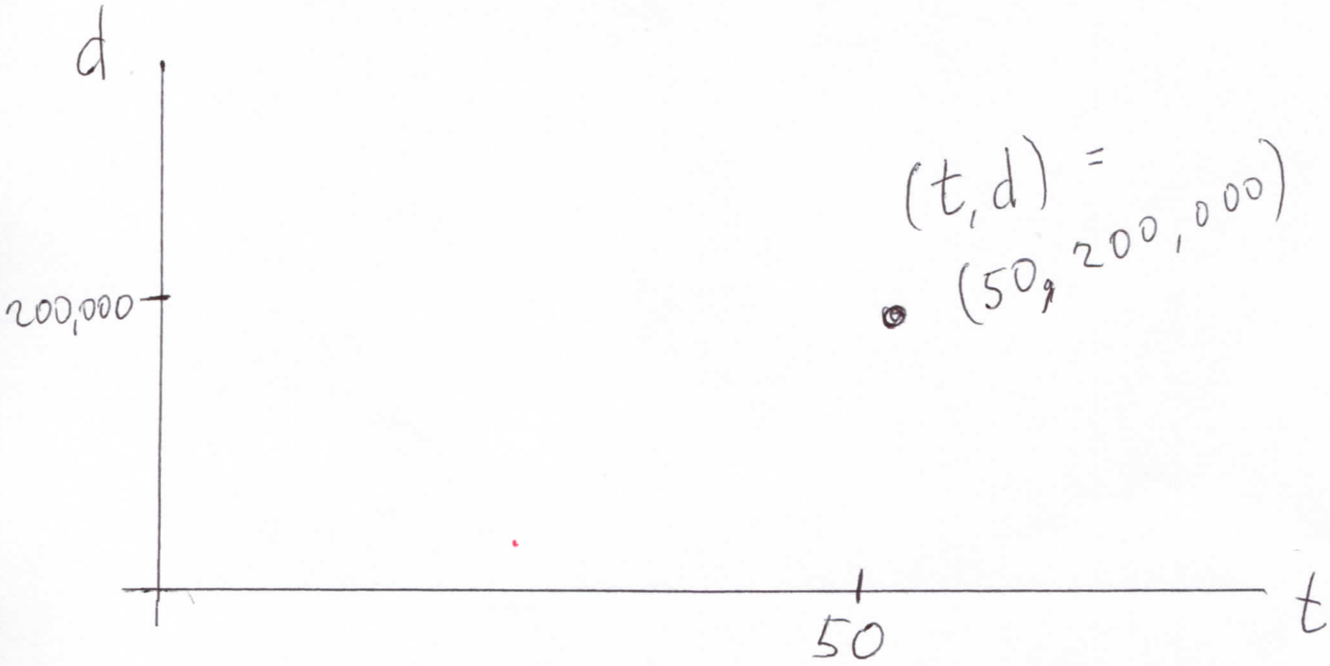
and by

$a(t)$ the acceleration of our spaceship pushing away from the sun, in meters per second squared

t seconds after leaving the earth.

Just by knowing when v is positive and a is positive, we will draw approximations of the graph of d, that will include the basic shape of the graph. "Graph" throughout this Magnification will mean a curve in the Cartesian plane with horizontal axis of nonnegative t values, vertical axis of nonnegative d values.

For example, if we are 200,000 meters from the sun 50 seconds after we left the earth, then our graph of d would include the ordered pair $(t, d) = (50, d(50)) = (50, 200, 000)$, as drawn below.



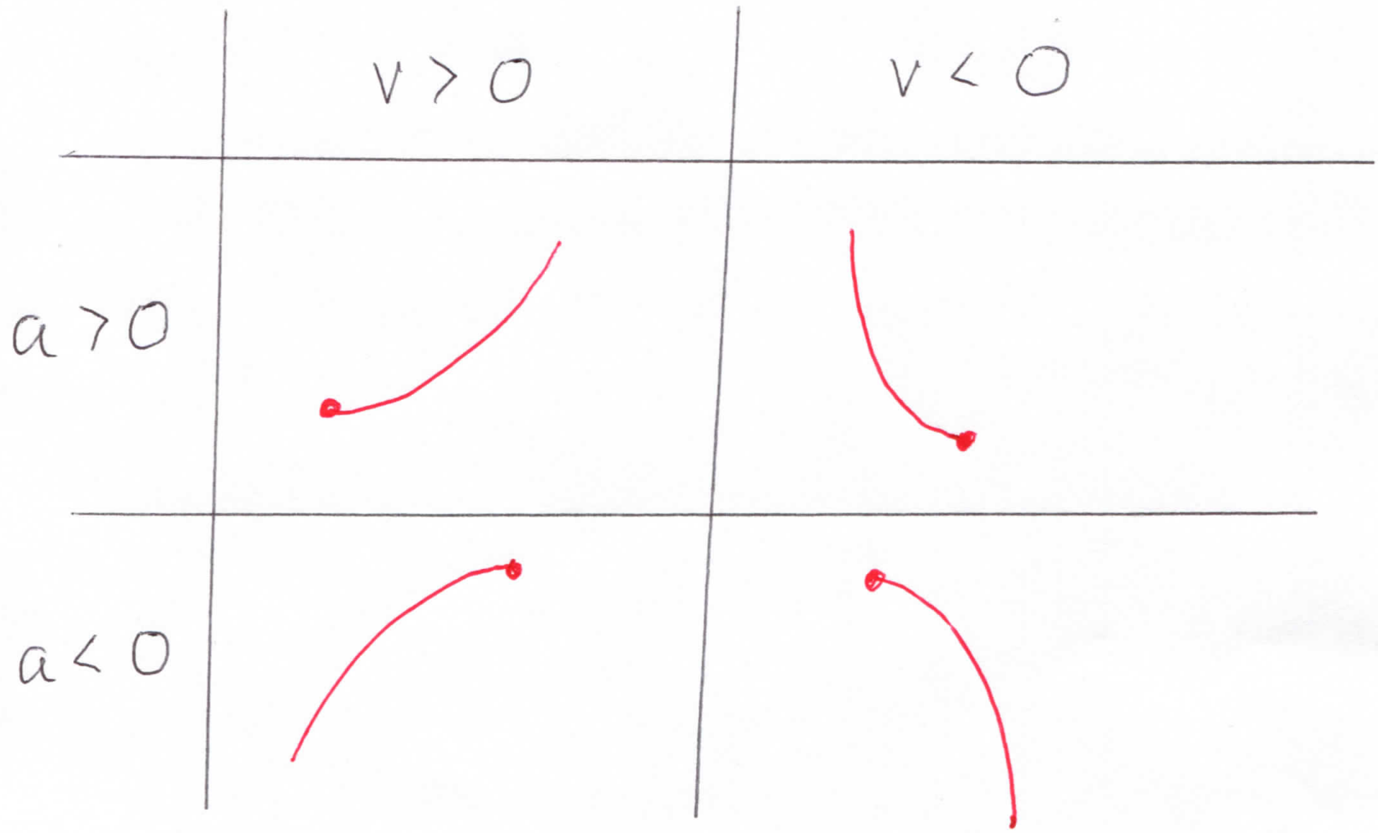
More specifically, "graph" will always mean a set of ordered pairs $\{(t, d(t))\}$, for $d(t)$ defined at the beginning of 1.1.

Distance is assumed to be *continuous*, meaning, informally, that its graph may be drawn in its entirety without lifting pen from paper.

It should also be noted that, if $d(t) = 0$ for some time $t \equiv t_0$, then the graph terminates because we have hit the sun and exploded; there will then be no graph for $t > t_0$.

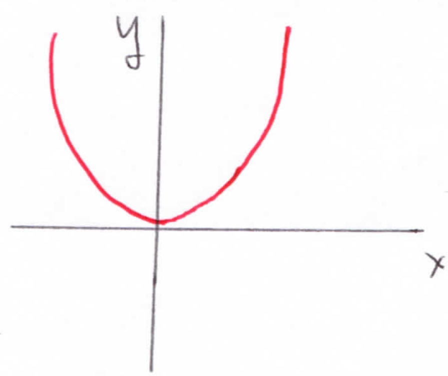
Our graphs will be entirely pasting together copies of the four curves in Table 1.2 on the next page.

TABLE 1.2

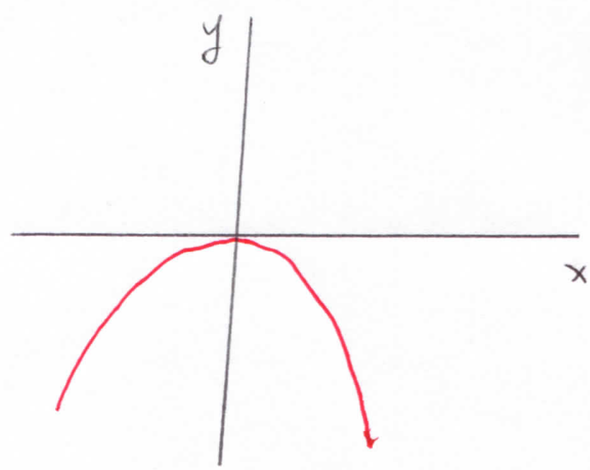


Remarks 1.3. The graphs in Table 1.2 are halves of what are called *parabolas* (see [2]). If the reader is armed with a graphing calculator, said reader may see the merging of the graphs in the first row $a > 0$ above by producing the graph of $y = x^2$ on a graphing calculator and may see the merging of the graphs in the second row $a < 0$ above by producing the graph of $y = -x^2$.

Proving the pictures in Table 1.2 requires calculus, so we will not go into it.



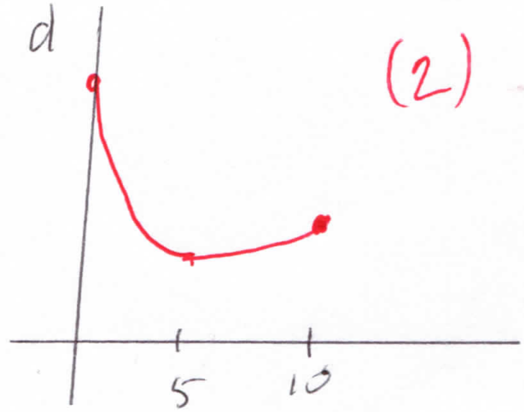
$y = x^2$



$y = -x^2$

Examples 1.4. The rough graphs we can create just by knowing where velocity and acceleration are positive and using Table 1.2 will give us clues about what values of t make $d(t)$ as large as possible (a *maximum*) and as small as possible (a *minimum*), that is, at what time we are furthest from the sun or closest to the sun.

For example, suppose we know that, for $0 < t < 5$, $v(t) < 0$ and $a(t) > 0$, while for $5 < t < 10$, $v(t) > 0$ and $a(t) > 0$. Either of the following graphs, labeled (1) and (2), satisfy those conditions. We see that $d(t)$ is smallest when $t = 5$; that is, we are closest to the sun when $t = 5$. But $d(t)$ is largest *either* when $t = 10$, as in graph (1), or when $t = 0$, as in graph (2).

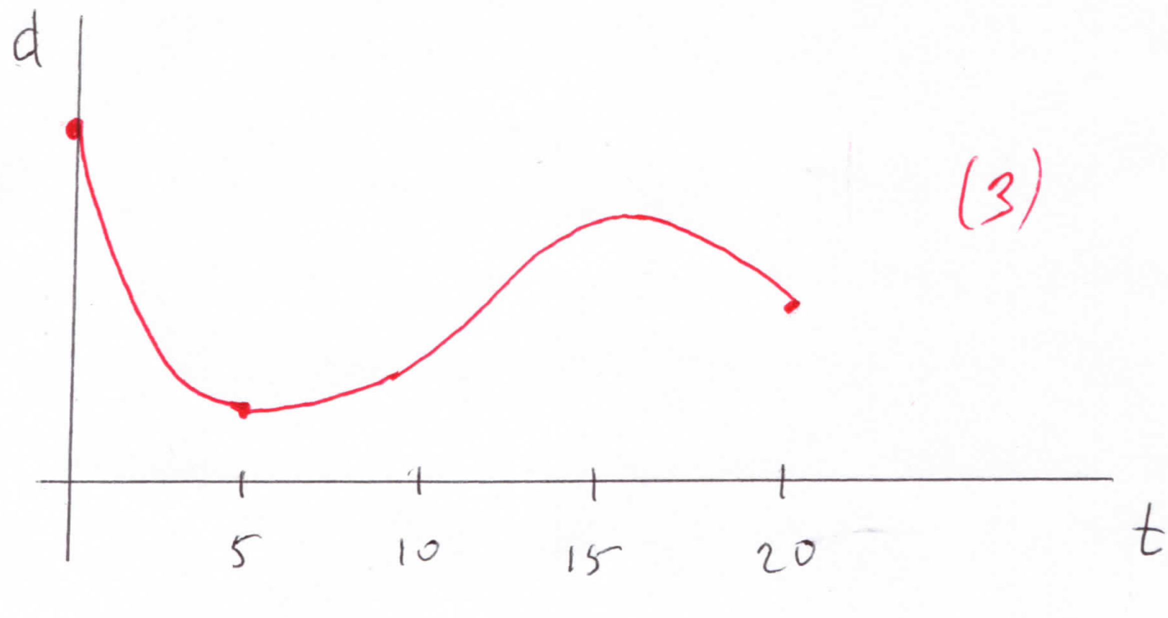


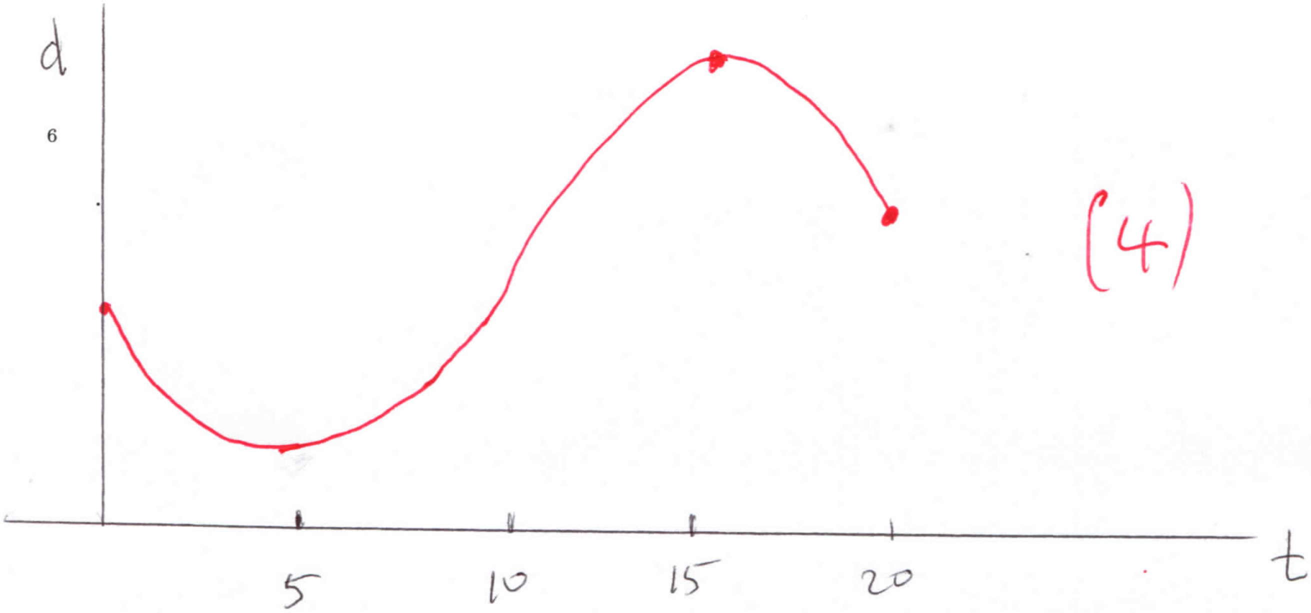
Thus the information about velocity and acceleration, in this case, although it guarantees that we are closest to the sun when $t = 5$, it only tells us that we are furthest from the sun either when $t = 0$ or $t = 5$; we cannot specify further.

Similarly, consider the conditions

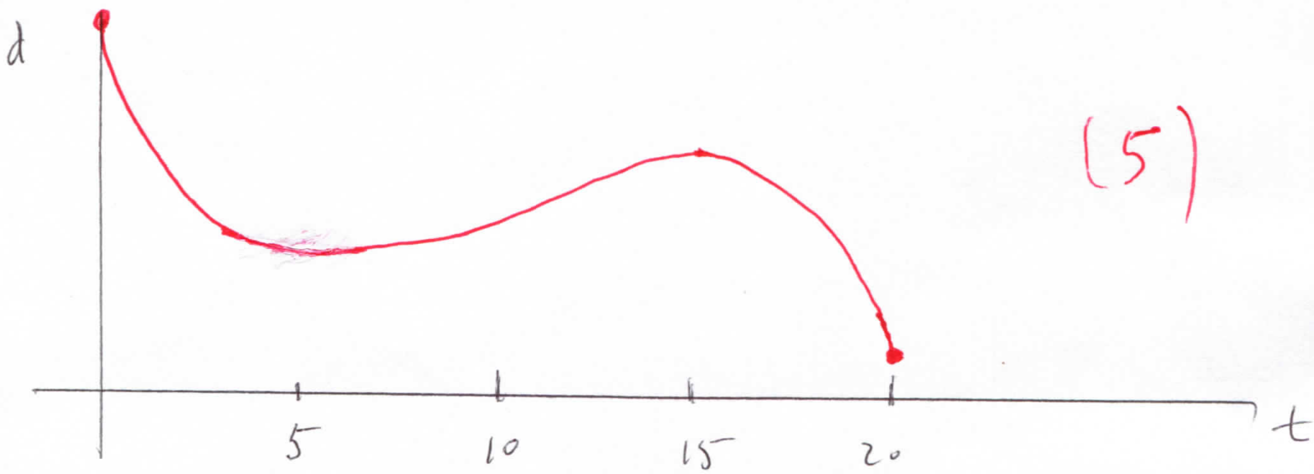
$$0 < t < 5 \rightarrow v(t) < 0 \text{ and } a(t) > 0, \;\; 5 < t < 10 \rightarrow v(t) > 0 \text{ and } a(t) > 0,$$
$$10 < t < 15 \rightarrow v(t) > 0 \text{ and } a(t) < 0, \;\; 15 < t < 20 \rightarrow v(t) < 0 \text{ and } a(t) < 0.$$

The following three graphs, labeled (3), (4), and (5), each satisfy the stated conditions. In graph (3), we are closest to the sun when $t = 5$ and furthest from the sun when $t = 0$; in graph (4), we are closest to the sun when $t = 5$ and furthest from the sun when $t = 15$; in graph (5), we are closest to the sun when $t = 20$ and furthest from the sun when $t = 0$;





(4)



(5)

For the information about velocity and acceleration realized in graphs (3), (4), and)5), the most we can say is that we are closest to the sun either when $t = 5$ or $t = 20$, and we are furthest from the sun either when $t = 0$ or $t = 15$.

The moral of graphs (1)–(5) is that the positivity or negativity of velocity and acceleration gives us the shape of the graph, except for vertical stretching or compressing. Among other things, it tells us where maxima or minima might occur, that is, when we might be furthest away from the sun or closest to the sun.

2. GRAPH of DISTANCE in FINITE TIME INTERVAL

In this section, we assume that everyone leaves the spaceship an hour after leaving the earth. Thus our graphs will only involve values of t less than or equal to 3600. We also assume that we do not crash into the sun.

Examples 2.1. In each part, draw a graph satisfying the specified conditions. To the extent possible, state when our spaceship is closest to the sun or farthest away from the sun; this might include visualizing vertical stretching or compressing, as in Examples 1.4.

(a) $v(t) > 0$ when $0 < t < 1200$;

$v(t) < 0$ when $1200 < t < 2400$ or $2400 < t < 3600$;

$a(t) > 0$ when $2400 < t < 3600$;

$a(t) < 0$ when $0 < t < 1200$ or $1200 < t < 2400$.

(b) $v(t) > 0$ when $0 < t < 1200$ or $1200 < t < 2400$ or $2400 < t < 3600$;

$v(t) < 0$ never;

$a(t) > 0$ when $0 < t < 1200$ or $2400 < t < 3600$;

$a(t) < 0$ when $1200 < t < 2400$.

(c) $v(t) > 0$ when $0 < t < 600$ or $2400 < t < 3600$;

$v(t) < 0$ when $600 < t < 1200$ or $1200 < t < 2400$;

$a(t) > 0$ when $1200 < t < 2400$ or $2400 < t < 3600$;

$a(t) < 0$ when $0 < t < 600$ or $600 < t < 1200$.

(d) $v(t) > 0$ when $0 < t < 600$ or $600 < t < 1800$;

$v(t) < 0$ when $1800 < t < 2400$ or $2400 < t < 3600$;

$a(t) > 0$ when $0 < t < 600$ or $2400 < t < 3600$;

$a(t) < 0$ when $600 < t < 1800$ or $1800 < t < 2400$.

(e) $v(t) > 0$ when $0 < t < 600$ or $600 < t < 1200$ or $2100 < t < 2400$ or $2400 < t < 3000$;

$v(t) < 0$ when $1200 < t < 1800$ or $1800 < t < 2100$ or $3000 < t < 3600$;

$a(t) > 0$ when $0 < t < 600$ or $1800 < t < 2100$ or $2100 < t < 2400$;

$a(t) < 0$ when $600 < t < 1200$ or $1200 < t < 1800$ or $2400 < t < 3000$ or $3000 < t < 3600$.

(f) $v(t) > 0$ when $600 < t < 1200$ or $1200 < t < 1800$;

$v(t) < 0$ when $0 < t < 600$ or $1800 < t < 2400$ or $2400 < t < 3600$;

$a(t) > 0$ when $0 < t < 600$ or $600 < t < 1200$ or $2400 < t < 3600$;

$a(t) < 0$ when $1200 < t < 1800$ or $1800 < t < 2400$.

(g) $v(t) > 0$ when $1200 < t < 1800$ or $1800 < t < 2400$;

$v(t) < 0$ when $0 < t < 600$ or $600 < t < 1200$ or $2400 < t < 3000$ or $3000 < t < 3600$;

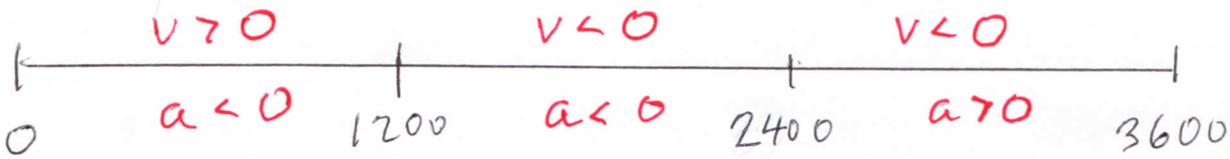
$a(t) > 0$ when $600 < t < 1200$ or $1200 < t < 1800$ or $3000 < t < 3600$;

$a(t) < 0$ when $0 < t < 600$ or $1800 < t < 2400$ or $2400 < t < 3000$.

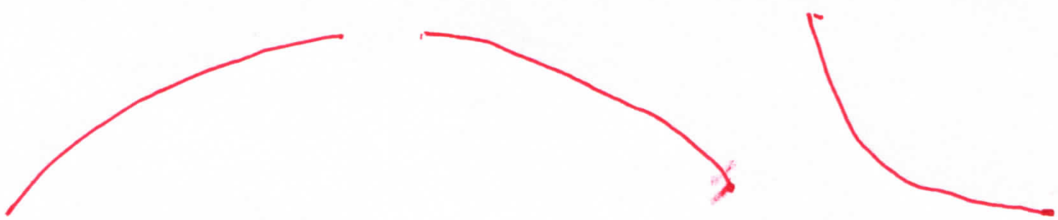
Examples 2.1 GRAPH SOLUTIONS.

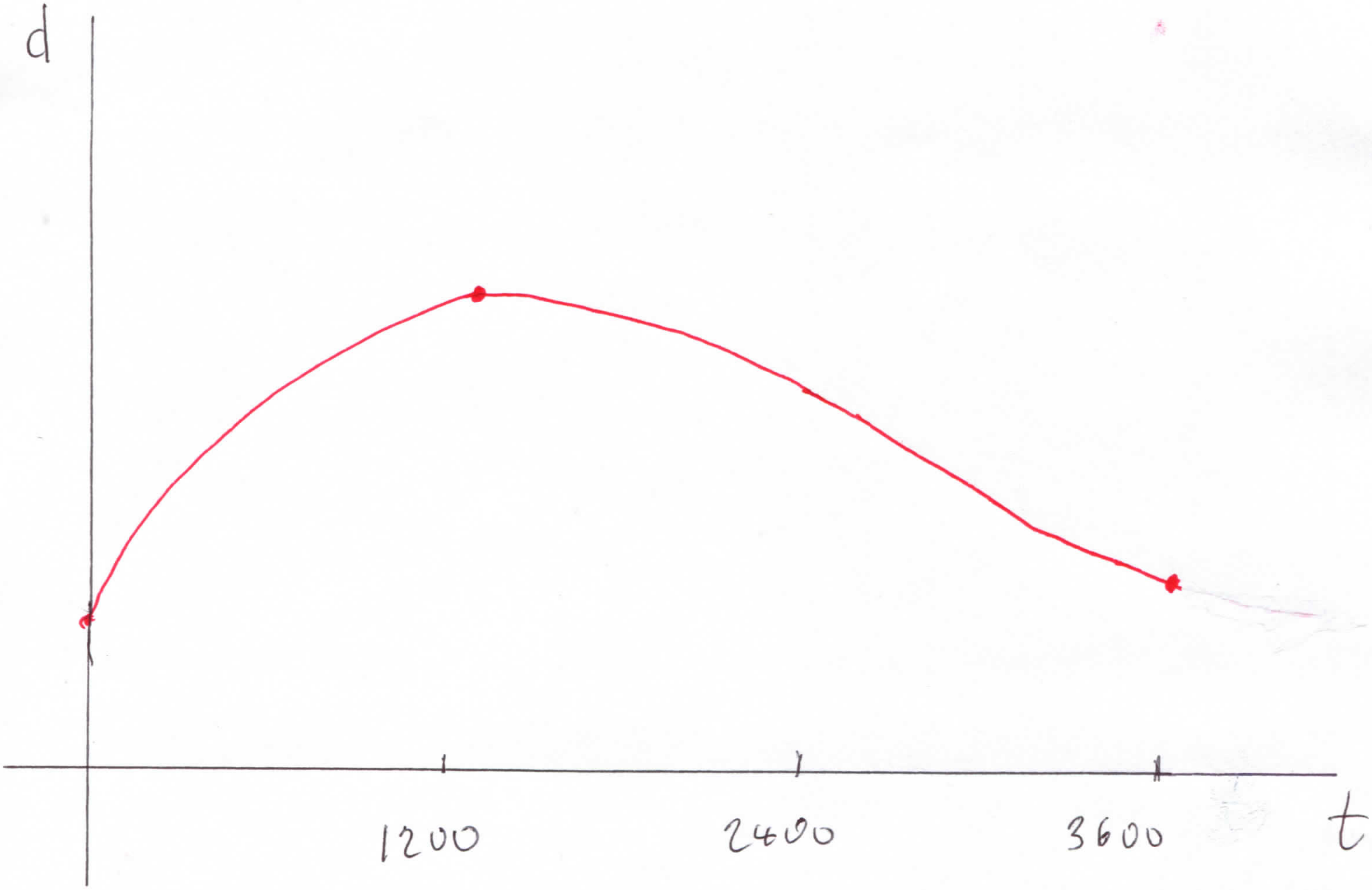
(a) Our conditions on velocity and acceleration may be organized as follows.

$0 < t < 1200 : v(t) > 0, a(t) < 0$; $1200 < t < 2400 : v(t) < 0, a(t) < 0$; $2400 < t < 3600 :$ $v(t) < 0, a(t) > 0$.

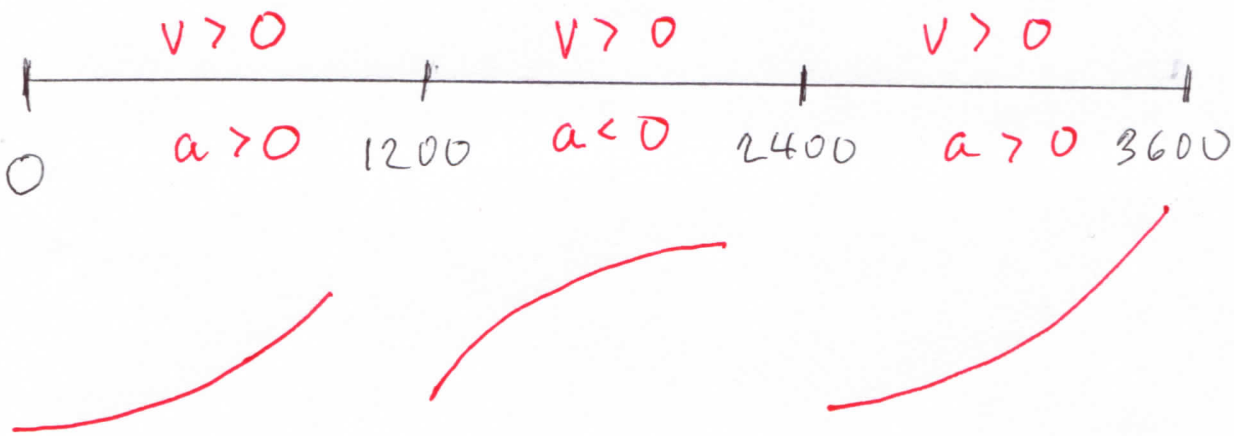


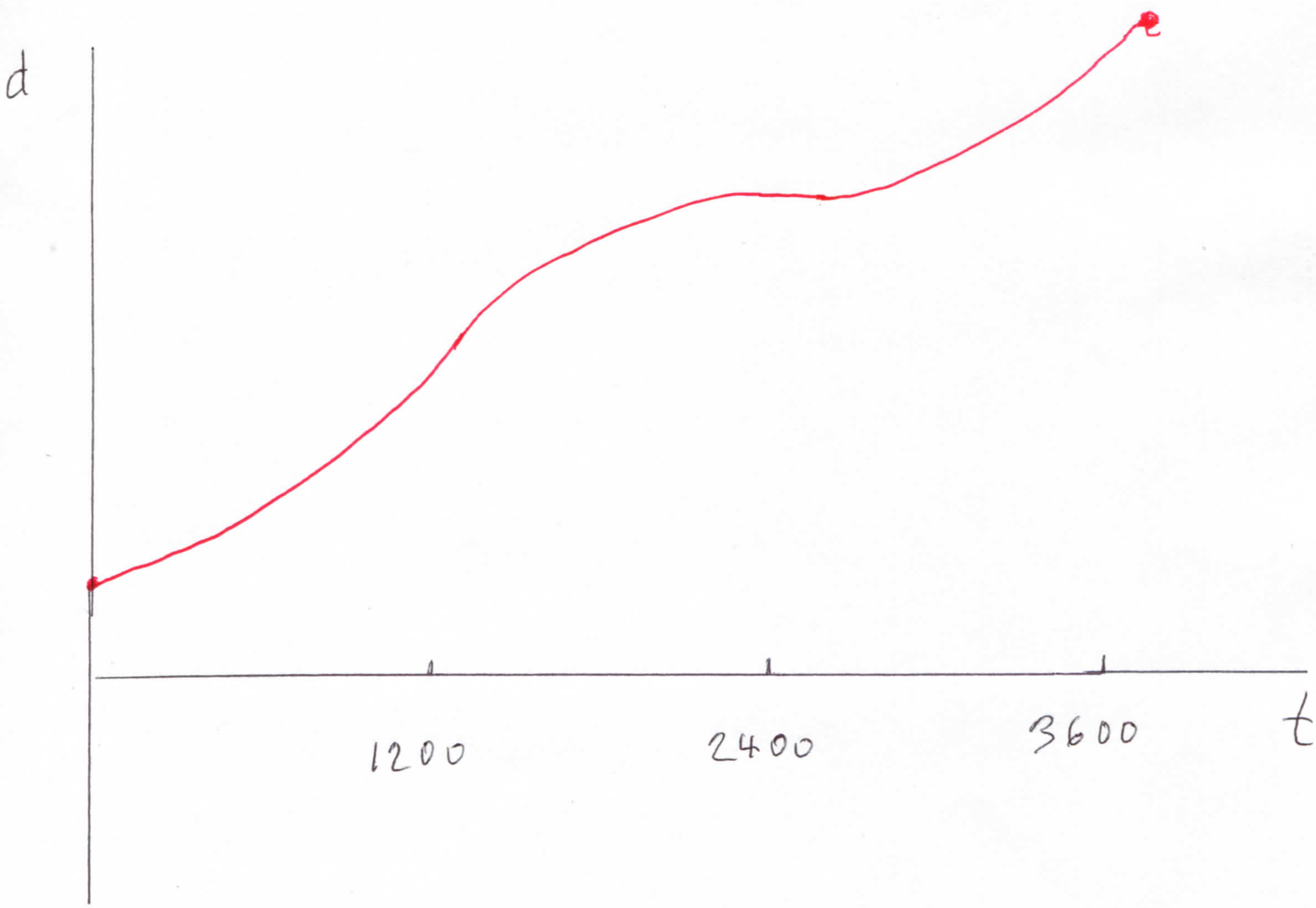
Thus we have the following three pictures from Table 1.2 to paste together:





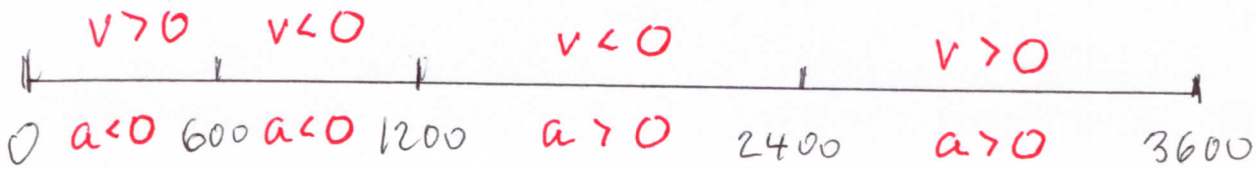
(b) Rewriting the conditions on velocity and acceleration, as in (a):
$0 < t < 1200 : v(t) > 0, a(t) > 0$; $1200 < t < 2400 : v(t) > 0, a(t) < 0$; $2400 < t < 3600 : v(t) > 0, a(t) > 0$,
leading to the three pictures from Table 1.2:

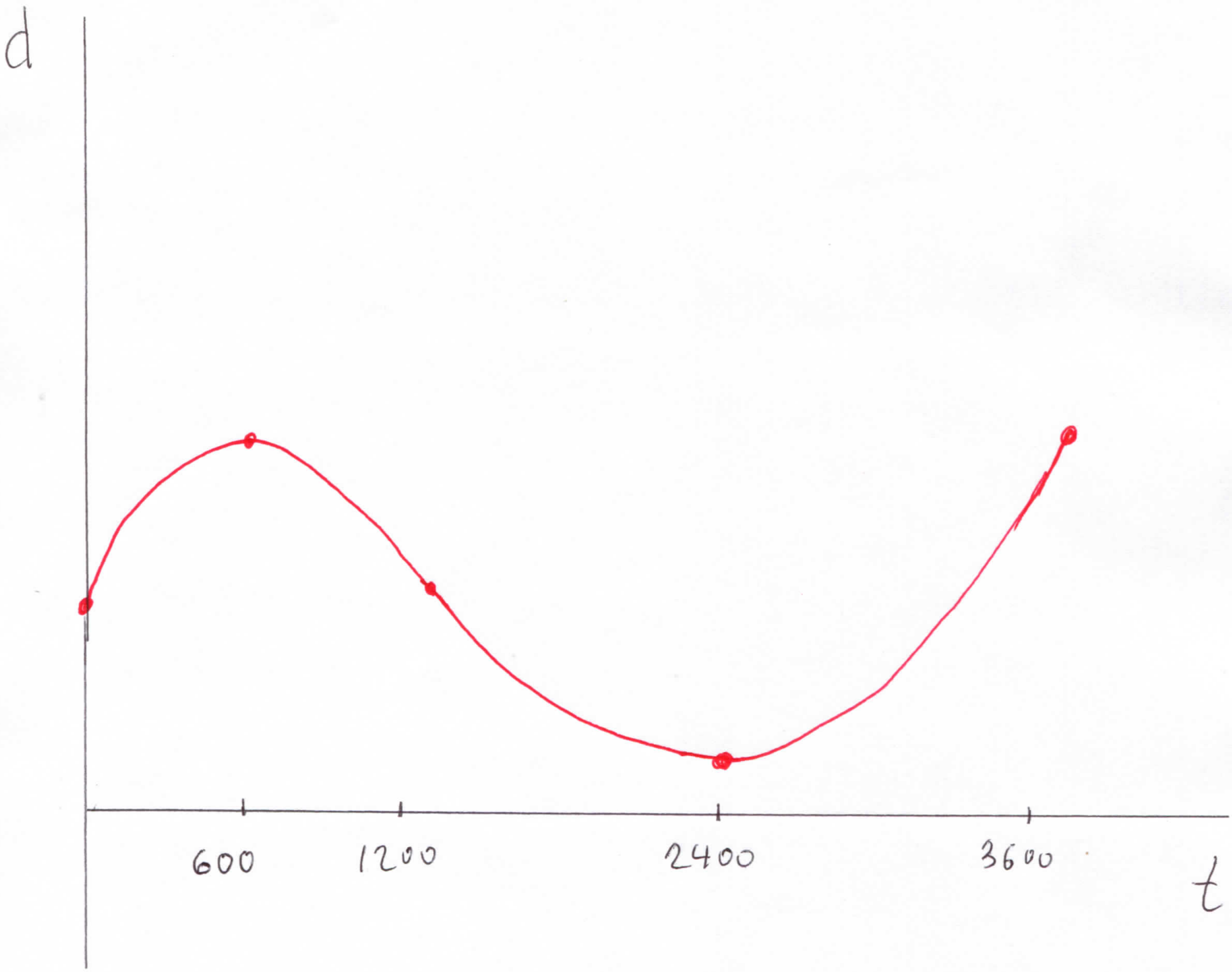




(c) Now we have four subintervals to focus on, leading to four pictures from Table 1.2 to mush together:

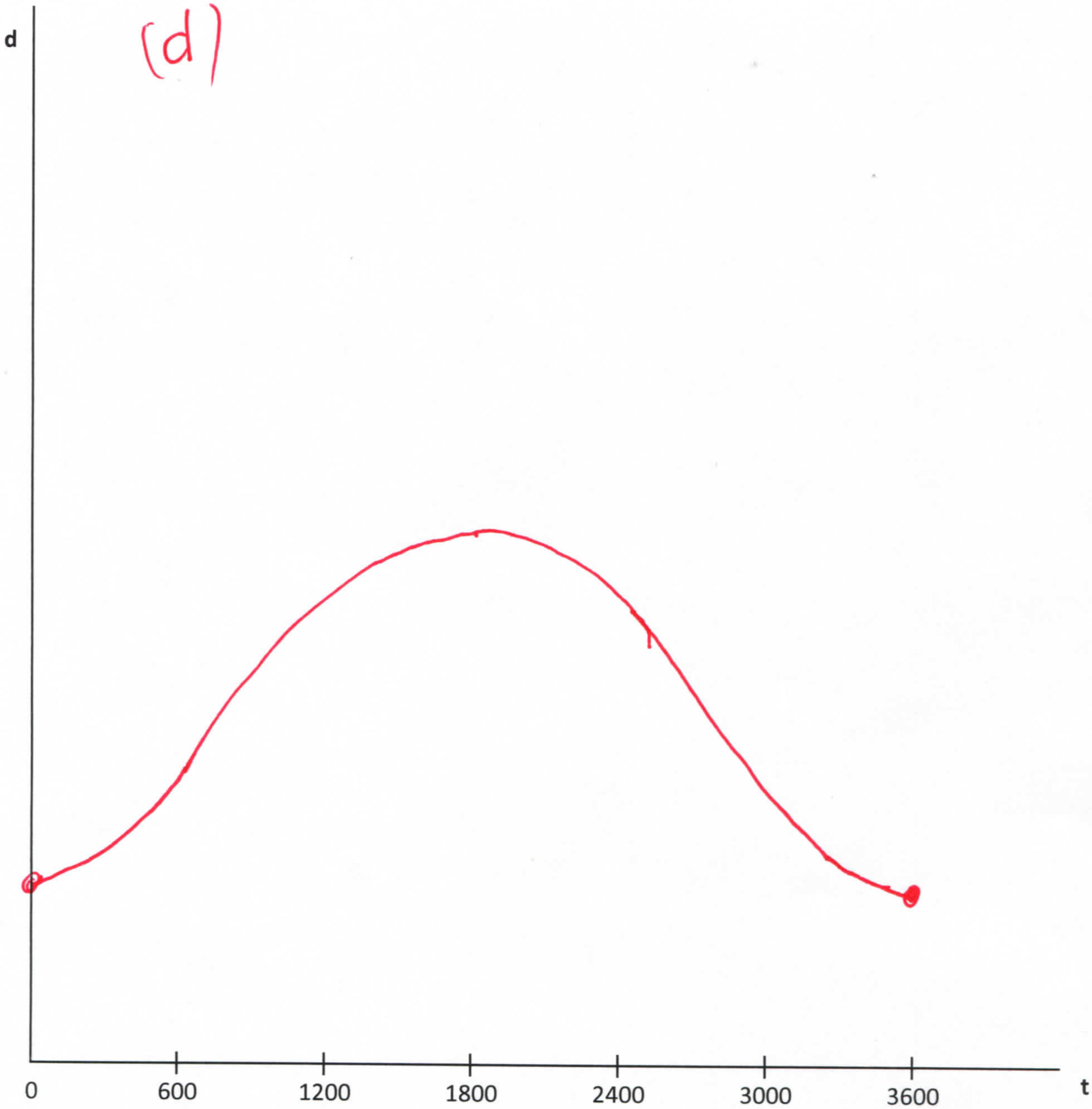
$$0 < t < 600 : v(t) > 0, a(t) < 0; \quad 600 < t < 1200 : v(t) < 0, a(t) < 0;$$
$$1200 < t < 2400 : v(t) < 0, a(t) > 0; \quad 2400 < t < 3600 : v(t) > 0, a(t) > 0.$$





(d)

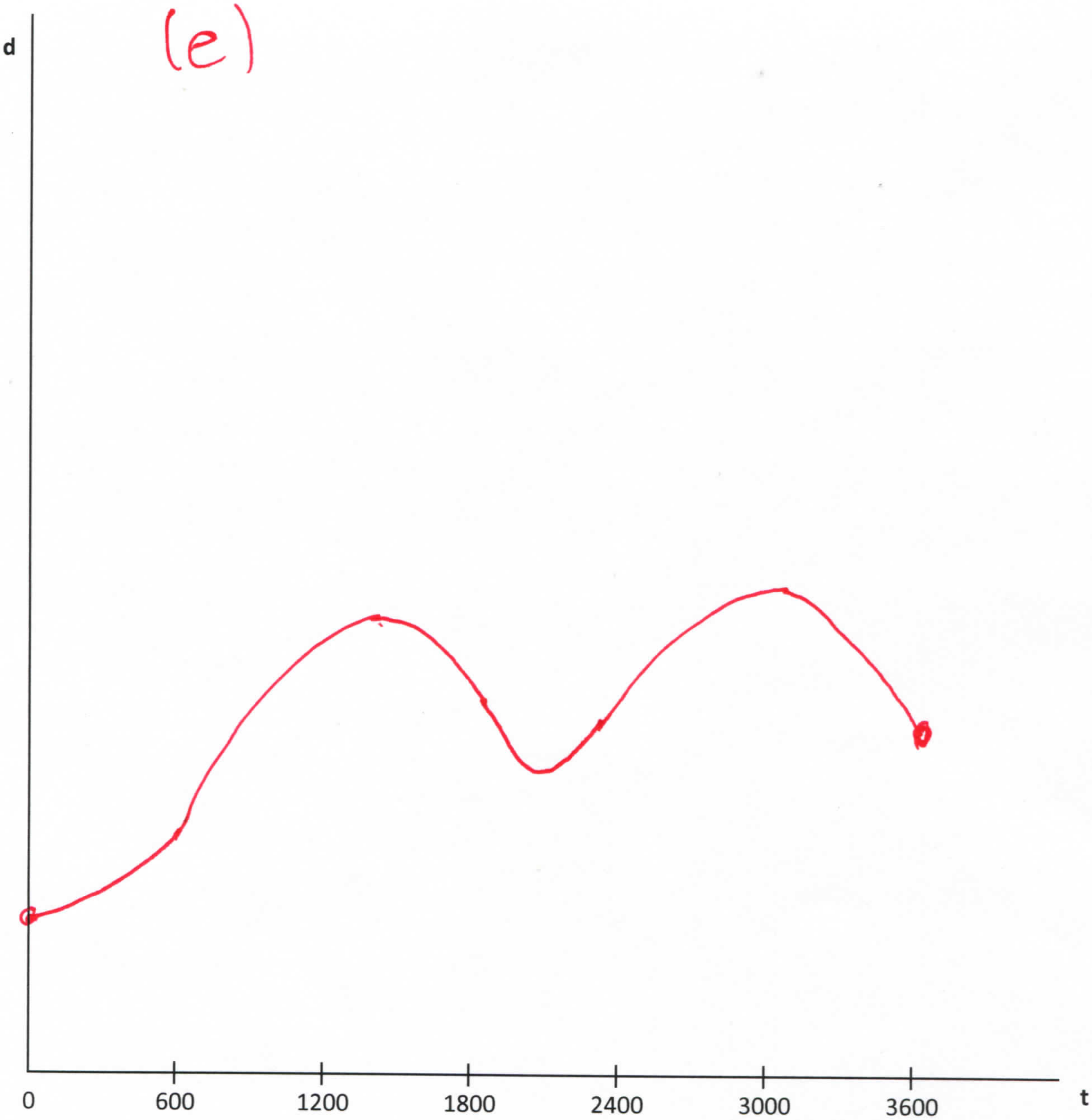
11

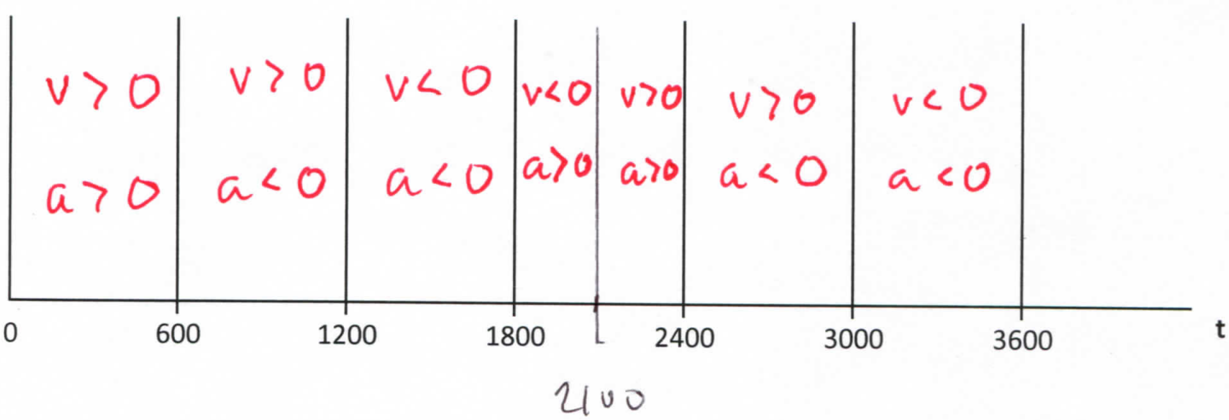


0–600	600–1200	1200–1800	1800–2400	2400–3000	3000–3600
$v > 0$	$v > 0$	$v > 0$	$v < 0$	$v < 0$	$v < 0$
$a > 0$	$a < 0$	$a < 0$	$a < 0$	$a > 0$	$a > 0$

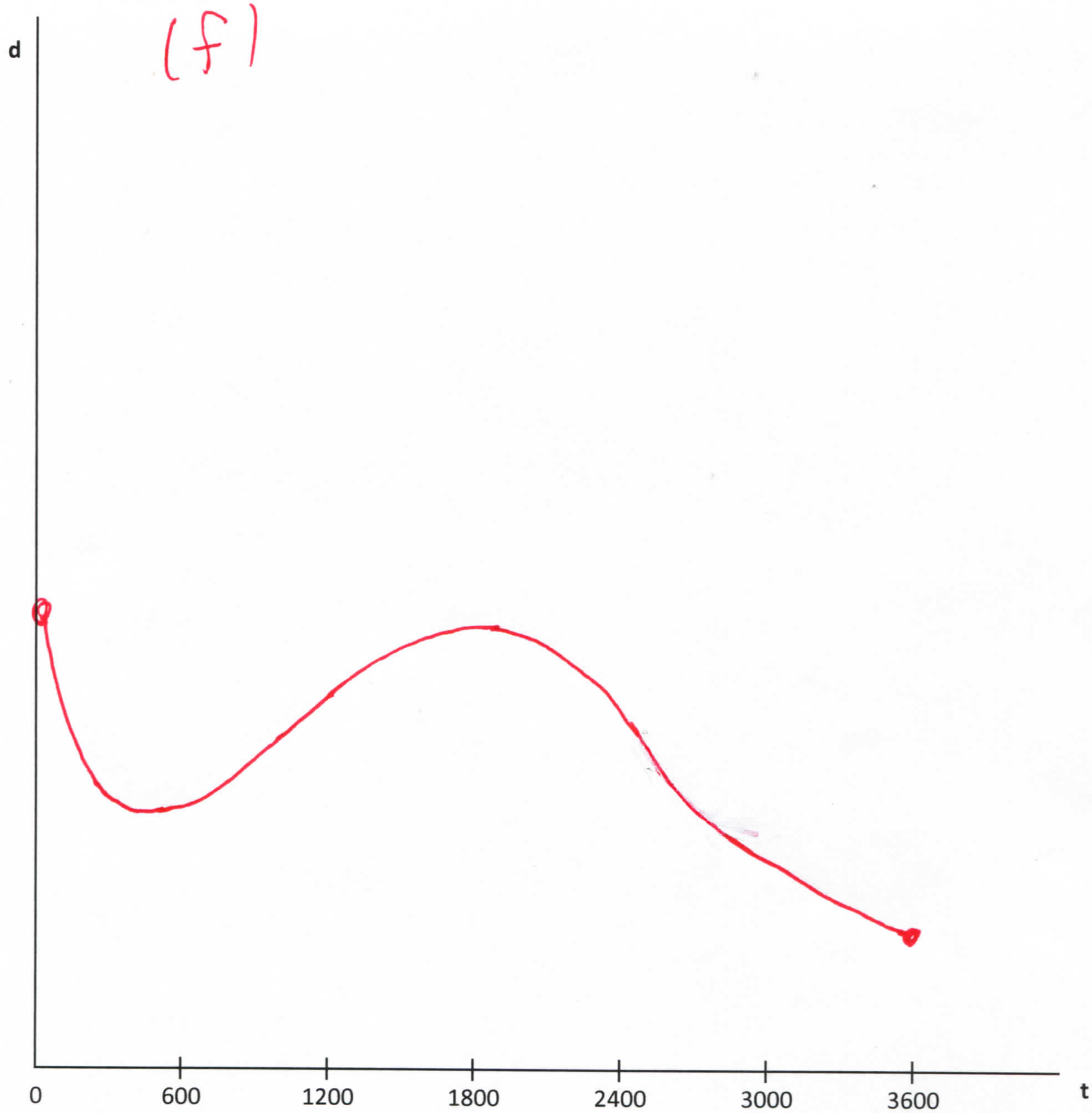
0 600 1200 1800 2400 3000 3600 t

(e)


d
0
600
1200
1800
2400
3000
3600
t


v > 0
a > 0
v > 0
a < 0
v < 0
a < 0
v<0
a>0
v>0
a>0
v > 0
a < 0
v < 0
a < 0
0
600
1200
1800
2400
3000
3600
t
2100

(f)

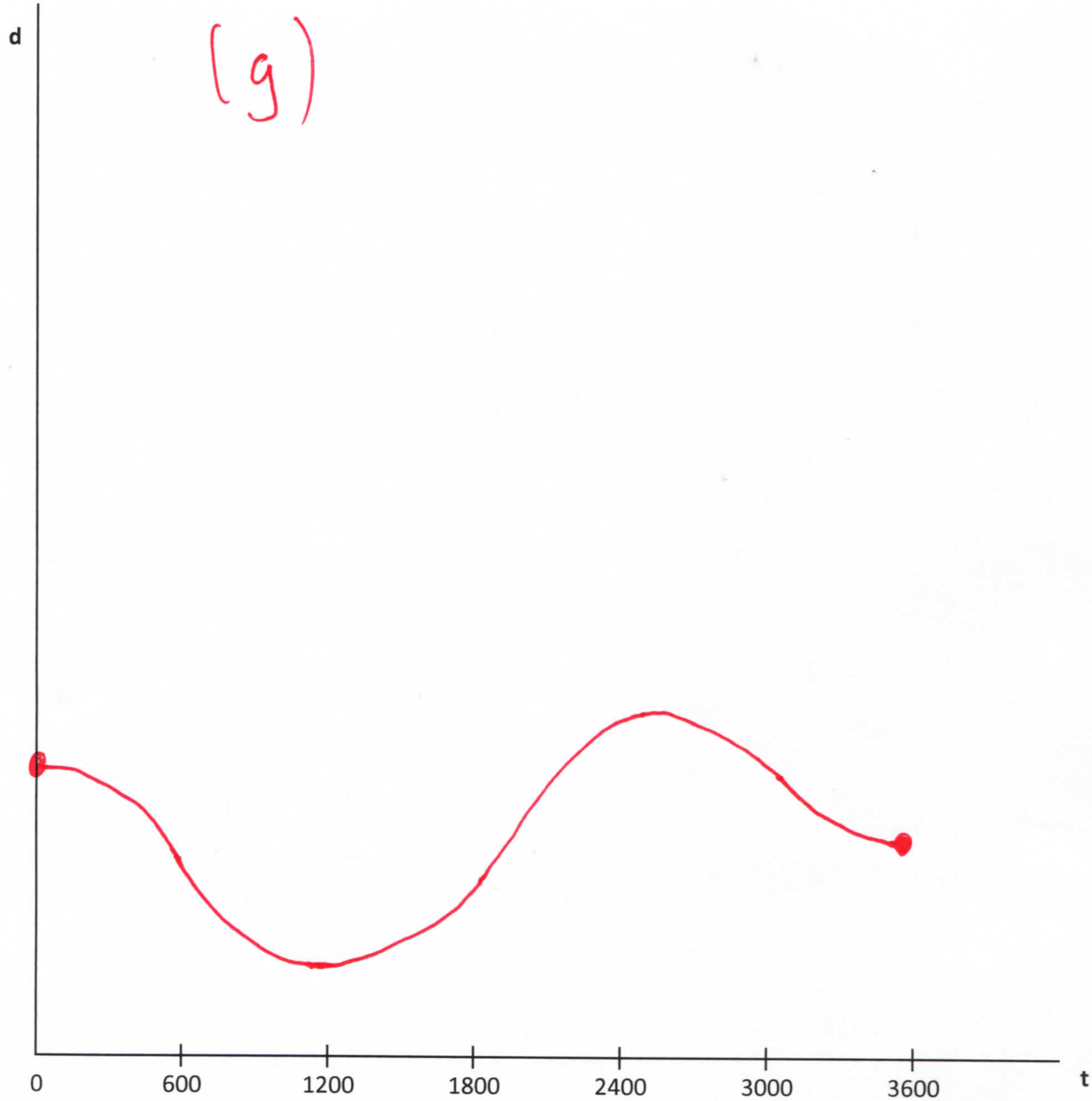


0–600	600–1200	1200–1800	1800–2400	2400–3000	3000–3600
$v<0$	$v>0$	$v>0$	$v<0$	$v<0$	$v<0$
$a>0$	$a>0$	$a<0$	$a<0$	$a>0$	$a>0$

0 600 1200 1800 2400 3000 3600 t

(g)

14



0–600	600–1200	1200–1800	1800–2400	2400–3000	3000–3600
v < 0	v < 0	v > 0	v > 0	v < 0	v < 0
a < 0	a > 0	a > 0	a < 0	a < 0	a > 0

0 600 1200 1800 2400 3000 3600 t

Examples 2.1 nonGRAPH SOLUTIONS

For each part, we will use our rough graph (keeping in mind how much variation in the graph is possible to still satisfy the specified conditions) to answer questions about distance to the sun.

(a) Our spaceship is farthest from the sun 1200 seconds (20 minutes) after leaving the earth. Our spaceship is closest to the sun either when we left the earth or 3600 seconds (1 hour) after leaving the earth.

(b) Our spaceship is farthest from the sun 3600 seconds (1 hour) after leaving the earth. Our spaceship is closest to the sun when we left the earth.

(c) Our spaceship is farthest from the sun either 600 seconds (10 minutes) after leaving the earth or 3600 seconds (1 hour) after leaving the earth. Our spaceship is closest to the sun either when we left the earth or 2400 seconds (40 minutes) after leaving the earth.

(d) Our spaceship is farthest from the sun 1800 seconds (30 minutes) after leaving the earth. Our spaceship is closest to the sun either when we left the earth or 3600 seconds (1 hour) after leaving the earth.

(e) Our spaceship is farthest from the sun either 1200 seconds (20 minutes) after leaving the earth or 3000 seconds (50 minutes) after leaving the earth. Our spaceship is closest to the sun either when we left the earth or 2100 seconds (35 minutes) after leaving the earth or 3600 seconds (1 hour) after leaving the earth.

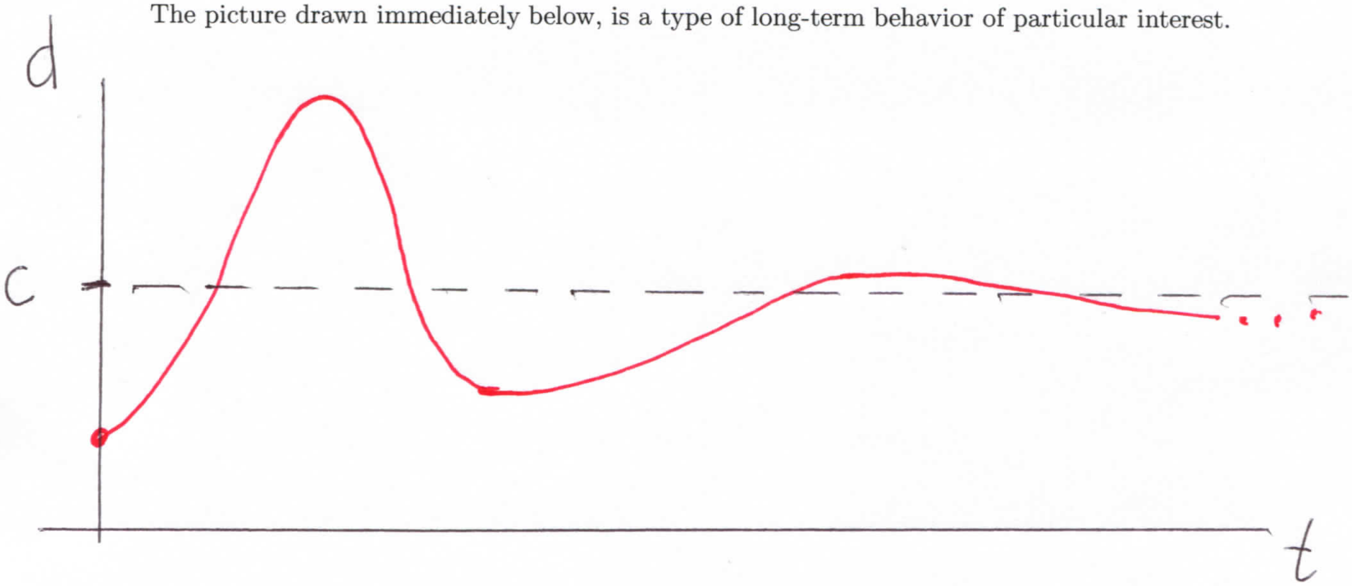
(f) Our spaceship is farthest from the sun either when we left the earth or 1800 seconds (30 minutes) after leaving the earth. Our spaceship is closest to the sun either 600 seconds (10 minutes) after leaving the earth or 3600 seconds (1 hour) after leaving the earth.

(g) Our spaceship is farthest from the sun either when we left the earth or 2400 seconds (40 minutes) after leaving the earth. Our spaceship is closest to the sun either 1200 seconds (20 minutes) after leaving the earth or 3600 seconds (1 hour) after leaving the earth.

3. LONG-TERM BEHAVIOR

"Long term" means lots of time goes by; we will assume in this section that no one ever leaves the spaceship.

The picture drawn immediately below, is a type of long-term behavior of particular interest.



In the drawing immediately above, the values of $d(t)$ can be forced to be arbitrarily close to the number c by making t sufficient large; $d = c$ (drawn as a dotted line) is then said to be a **horizontal asymptote** of the graph.

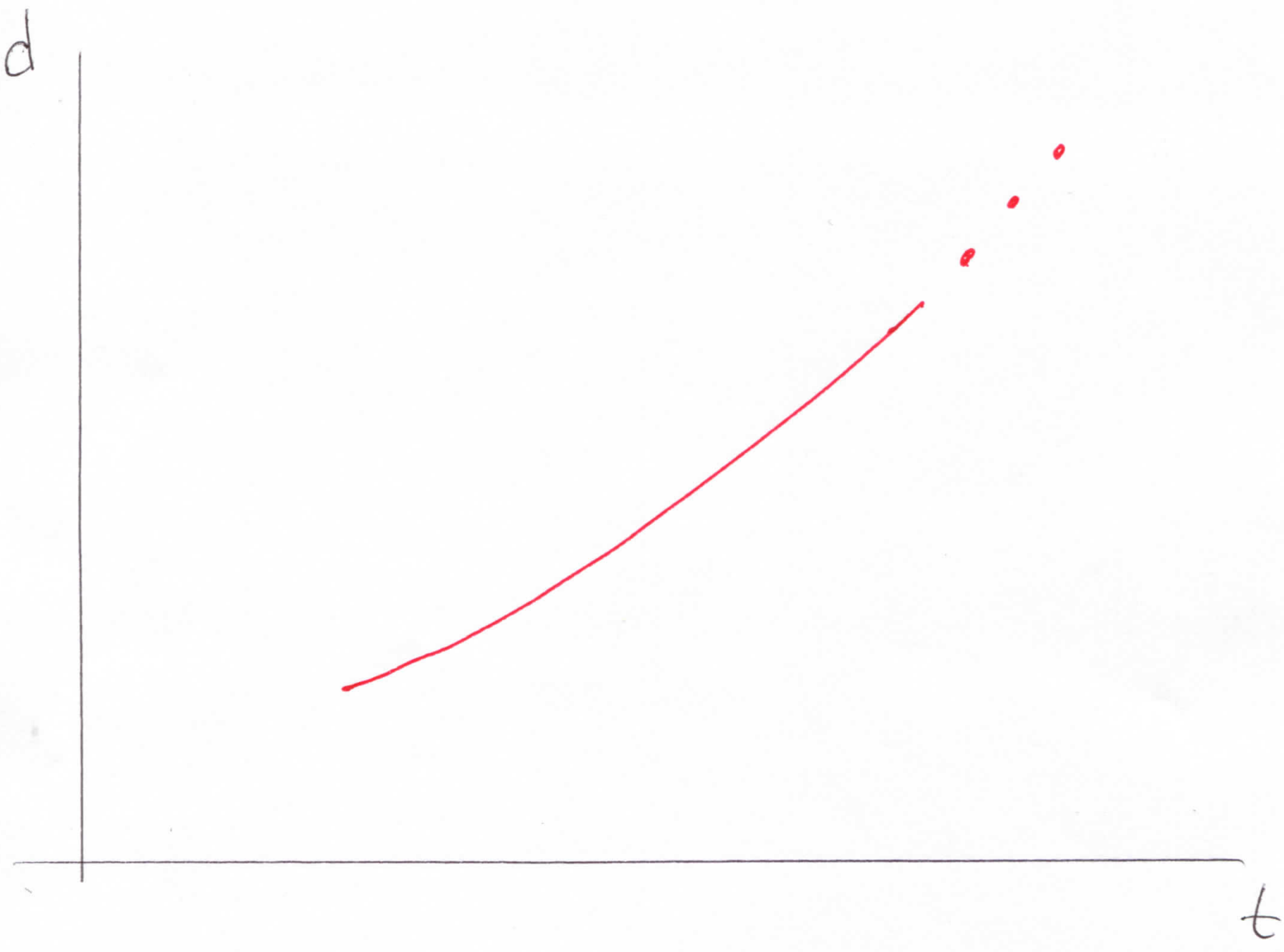
In terms of the spaceship of this Magnification, we are getting arbitrarily close to the fixed point c meters away from the sun on our straight line of motion. This point is an *equilibrium point.*

In the language of calculus, $c = \lim_{t\to\infty} d(t)$, shorthand for "the limit, as t goes to infinity, of $d(t)$ is c."

It is often the case that a physical system is chaotic and confusing in the short term, but quickly settles down close to an equilibrium. For example, suppose a box had an impermeable barrier separating two halves, with air in one half and a vacuum in the other half. If we removed the barrier, the densities of air at different points in the box would briefly be quite confusing, but it would soon get close to its equilibrium of uniform density throughout the box. It is very convenient, in this sort of situation, to approximate physical states with the equilibrium state.

Let's look at the possible graphs if we have one of the four pictures in Table 1.2 as t gets large.

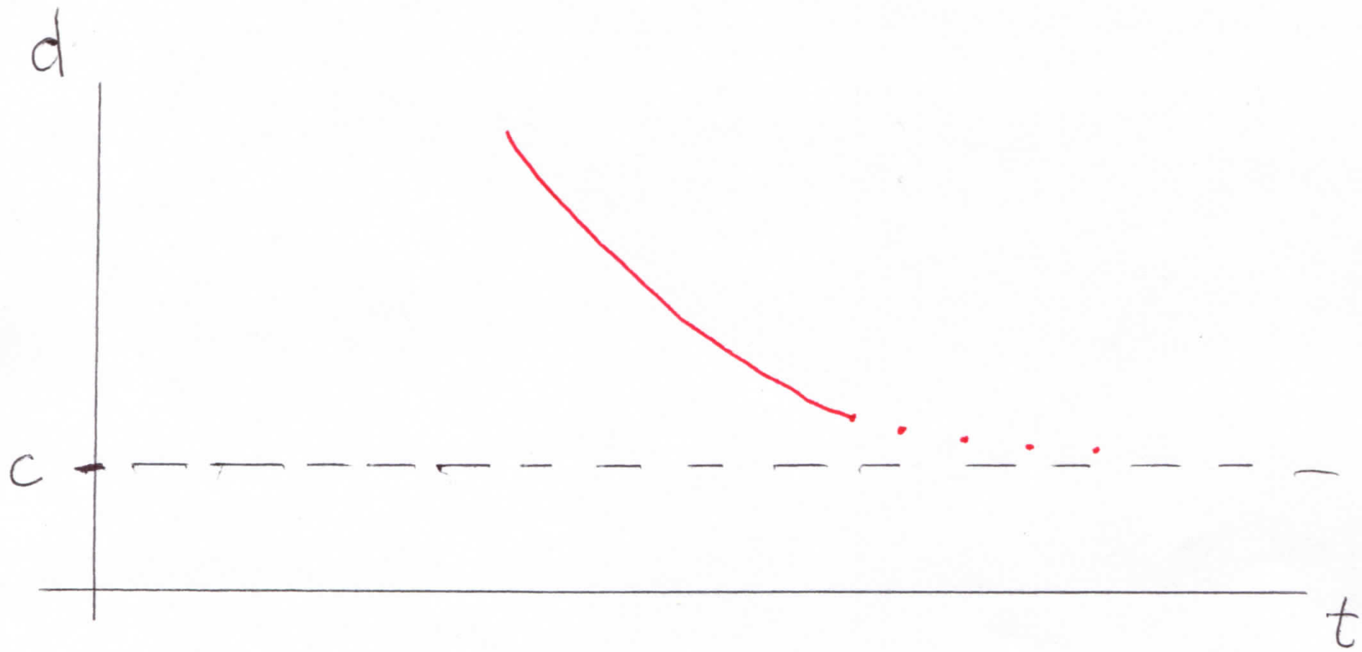
Long-Term Picture 3.1. $v(t) > 0$ and $a(t) > 0$ for t sufficiently large: Only one shape is possible, with no horizontal asymptote. The values of $d(t)$ are forced to be arbitrarily large, as t gets large.

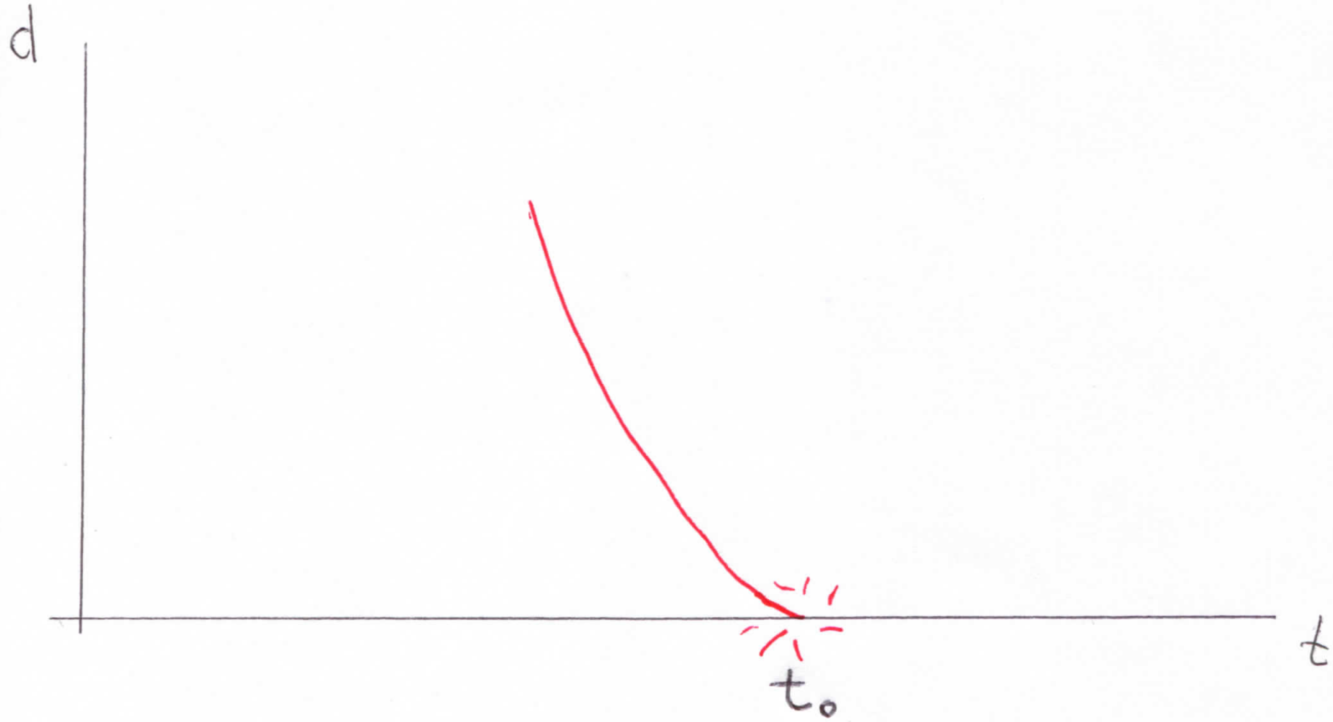


Long-Term Pictures 3.2. $v(t) < 0$ and $a(t) > 0$ for t sufficiently large: Two shapes are possible, depending on whether $d(t)$ hits zero.

If $d(t)$ never equals zero, we get a horizontal asymptote, as in the first drawing below.

If $d(t_0) = 0$, for some $t_0 > 0$, then the graph terminates (the space ship is consumed by the sun), with nothing for $t > t_0$, as in the second drawing below (no horizontal asymptote).



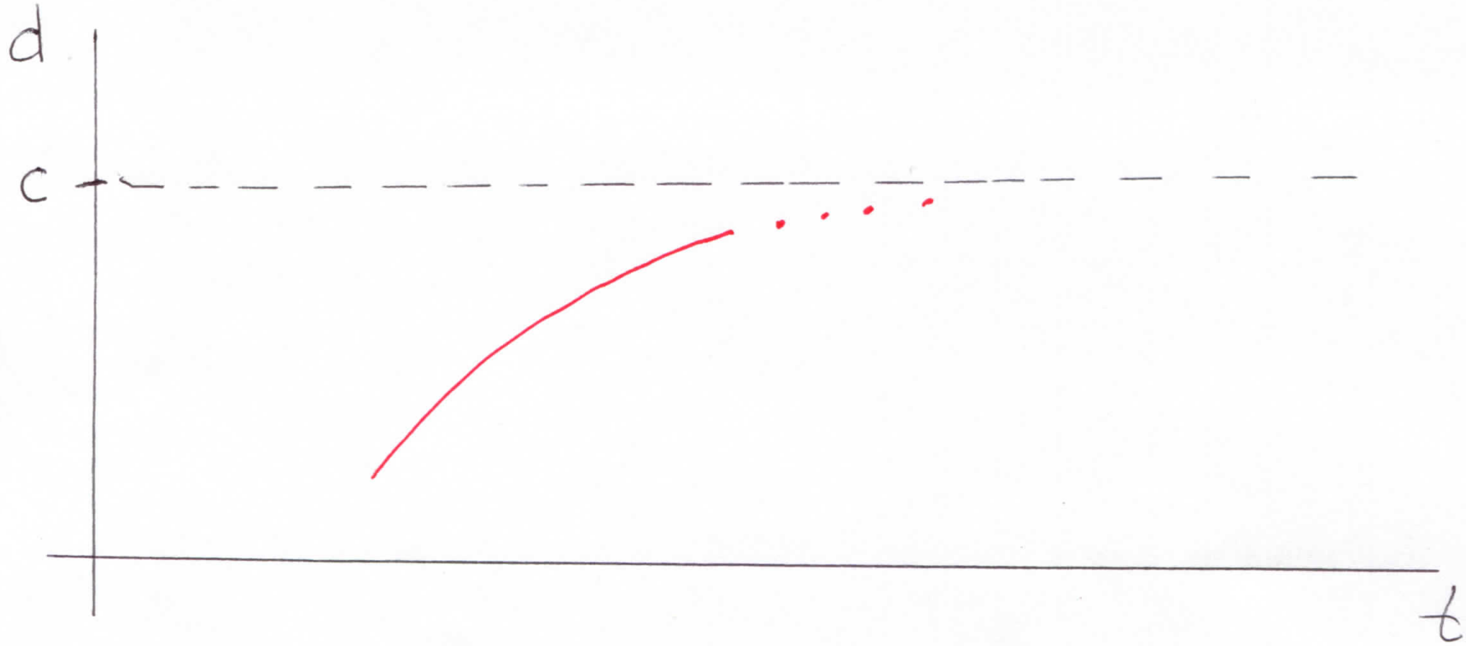


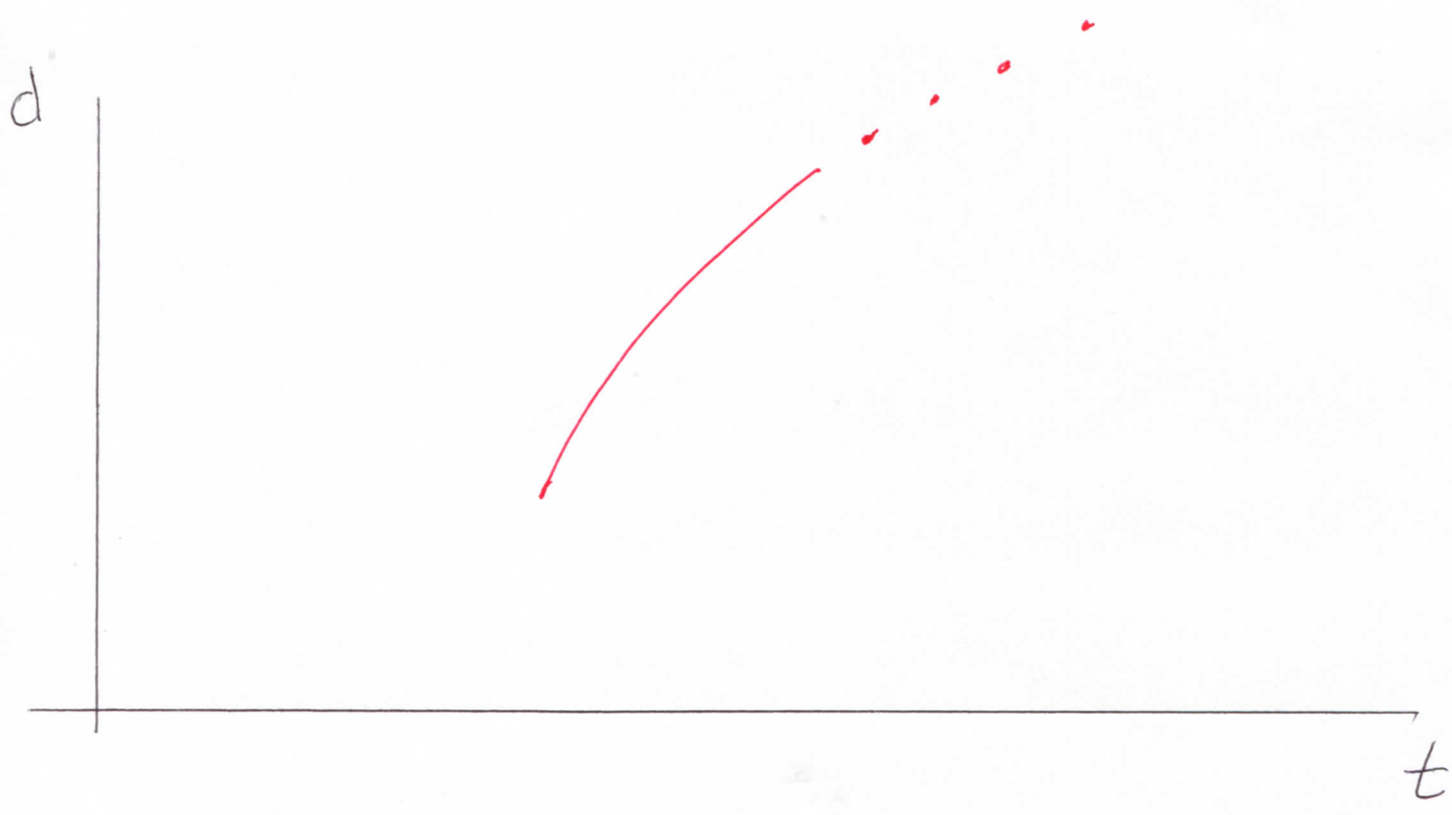
Long-Term Pictures 3.3. $v(t) > 0$ and $a(t) < 0$ for t sufficiently large: Two shapes are possible.

If there is a number that $d(t)$ never surpasses, we will have a horizontal asymptote, as in the first drawing below.

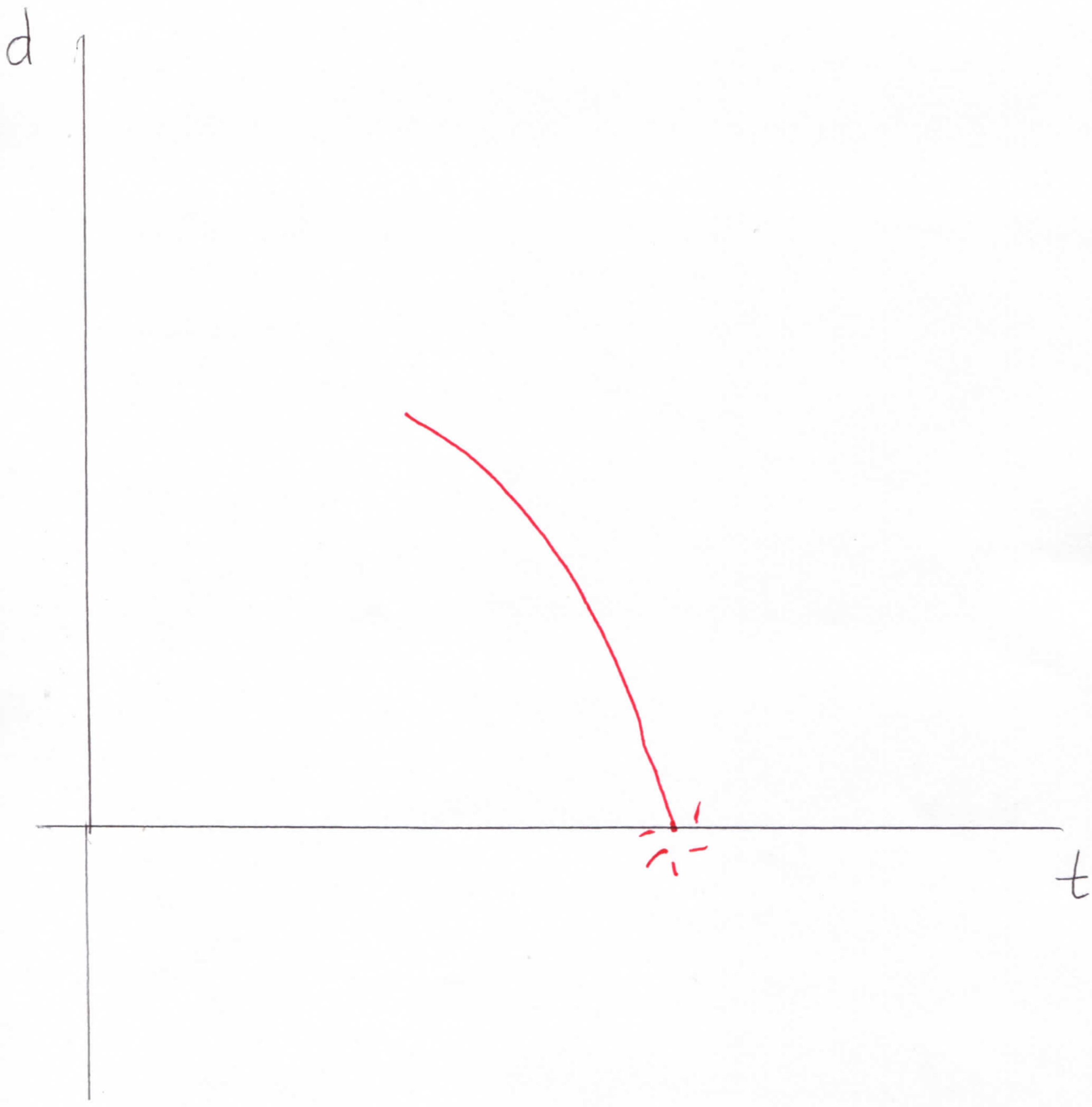
It is also possible to have the values of $d(t)$ forced to be arbitrarily large as t gets large, as in Long-Term Picture 3.1. See the second drawing below (no horizontal asymptote) and compare it to Long-Term Picture 3.1.

It can be shown (with calculus) that $d(t) \equiv \sqrt{t}$ is an example of the second drawing below.





Long-Term Picture 3.4. $v(t) < 0$ and $a(t) < 0$ for t sufficiently large: This one is doomed to fall into the sun, as drawn below. This means no horizontal asymptote, with the graph terminating (where the spaceship falls into the sun).



Examples 3.5. In each part, draw all variations, relevant to the questions that follow, of graphs satisfying the specified conditions. To the extent possible, state when our spaceship is closest to the sun or farthest away from the sun; this might include visualizing vertical stretching or compressing, as in Examples 1.4. If we get arbitrarily far away from the sun, by letting sufficient time pass, this should be mentioned. If we hit the sun, this should be mentioned. Any possible asymptotes should be mentioned and drawn as a dotted line.

Assume that any sun crashing is after $t = 3600$.

(a) See Examples 2.1(a).

$v(t) > 0$ when $0 < t < 1200$;

$v(t) < 0$ when $1200 < t < 2400$ or $2400 < t < 3600$ or $t > 3600$ (at least until we hit the sun, if we hit the sun);

$a(t) > 0$ when $2400 < t < 3600$ or $t > 3600$ (at least until we hit the sun, if we hit the sun);

$a(t) < 0$ when $0 < t < 1200$ or $1200 < t < 2400$.

(b) See Examples 2.1(a).

$v(t) > 0$ when $0 < t < 1200$ or $t > 3600$;

$v(t) < 0$ when $1200 < t < 2400$ or $2400 < t < 3600$;

$a(t) > 0$ when $2400 < t < 3600$ or $t > 3600$;

$a(t) < 0$ when $0 < t < 1200$ or $1200 < t < 2400$.

(c) See Examples 2.1(a).

$v(t) > 0$ when $0 < t < 1200$;

$v(t) < 0$ when $1200 < t < 2400$ or $2400 < t < 3600$ or $t > 3600$ (at least until we hit the sun);

$a(t) > 0$ when $2400 < t < 3600$;

$a(t) < 0$ when $0 < t < 1200$ or $1200 < t < 2400$ or $t > 3600$ (at least until we hit the sun).

(d) See Examples 2.1(b).

$v(t) > 0$ when $0 < t < 1200$ or $1200 < t < 2400$ or $2400 < t < 3600$ or $t > 3600$;

$v(t) < 0$ never;

$a(t) > 0$ when $0 < t < 1200$ or $2400 < t < 3600$;

$a(t) < 0$ when $1200 < t < 2400$ or $t > 3600$.

Examples 3.5 GRAPH SOLUTIONS.

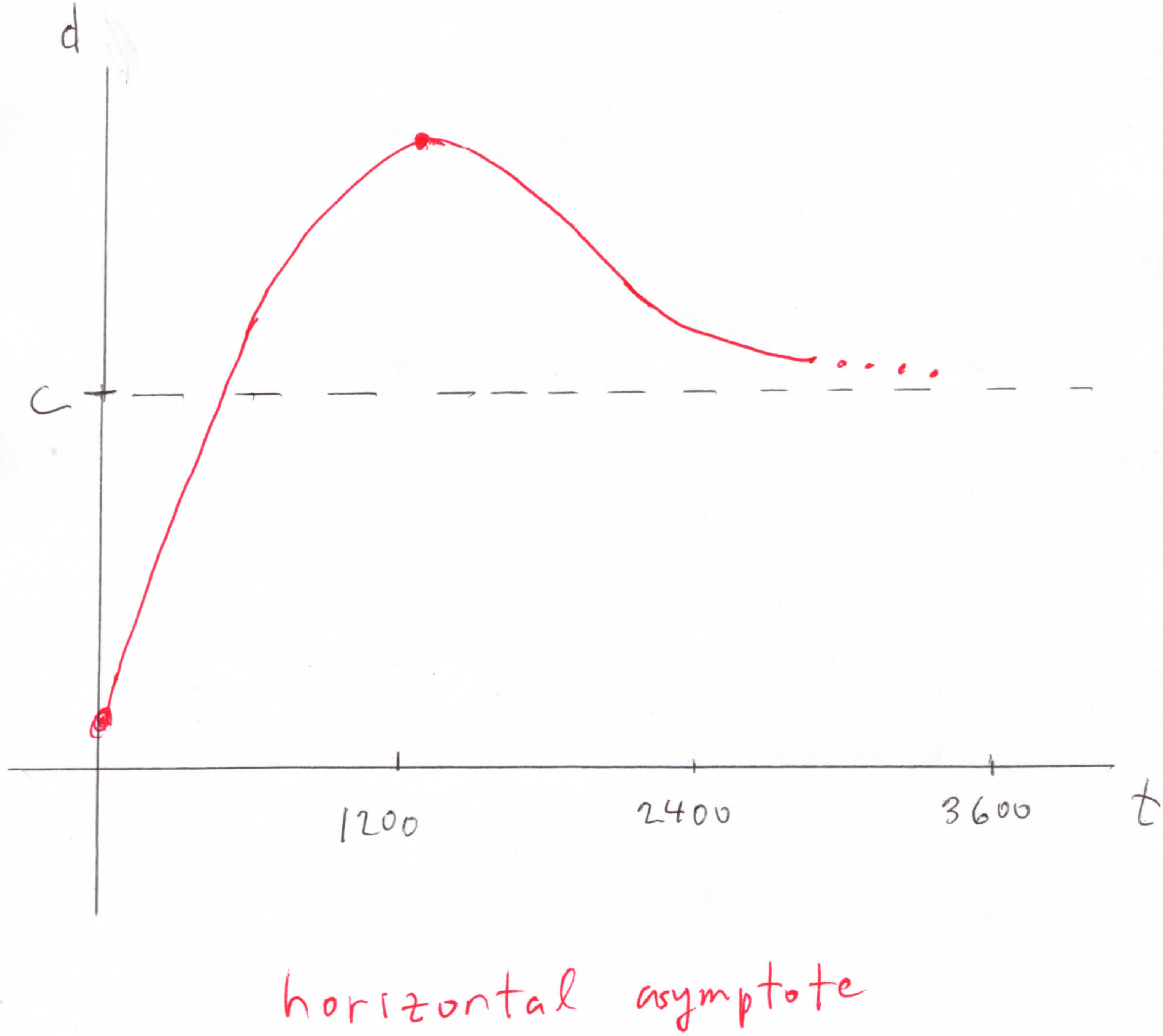
(a) As in Examples 2.1(a), reorganize:

$$0 < t < 1200 : v(t) > 0, a(t) < 0; \quad 1200 < t < 2400 : v(t) < 0, a(t) < 0;$$

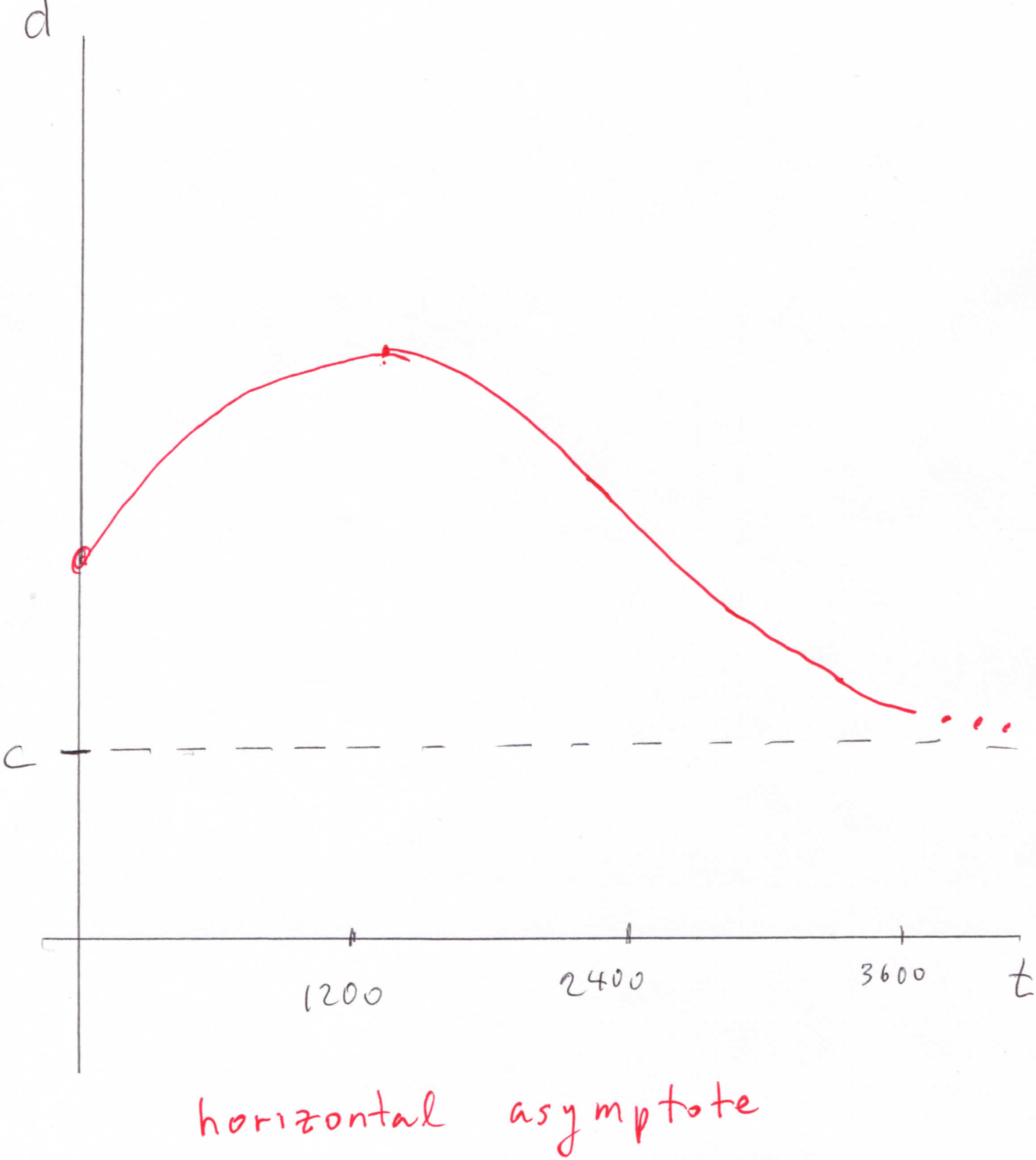
$$2400 < t < 3600 : \quad v(t) < 0, a(t) > 0; \quad t > 3600 : \quad v(t) < 0, a(t) > 0.$$

Alternatively, we could take the graph from Examples 2.1(a), and add on $v(t) < 0, a(t) > 0$, for $t > 3600$.

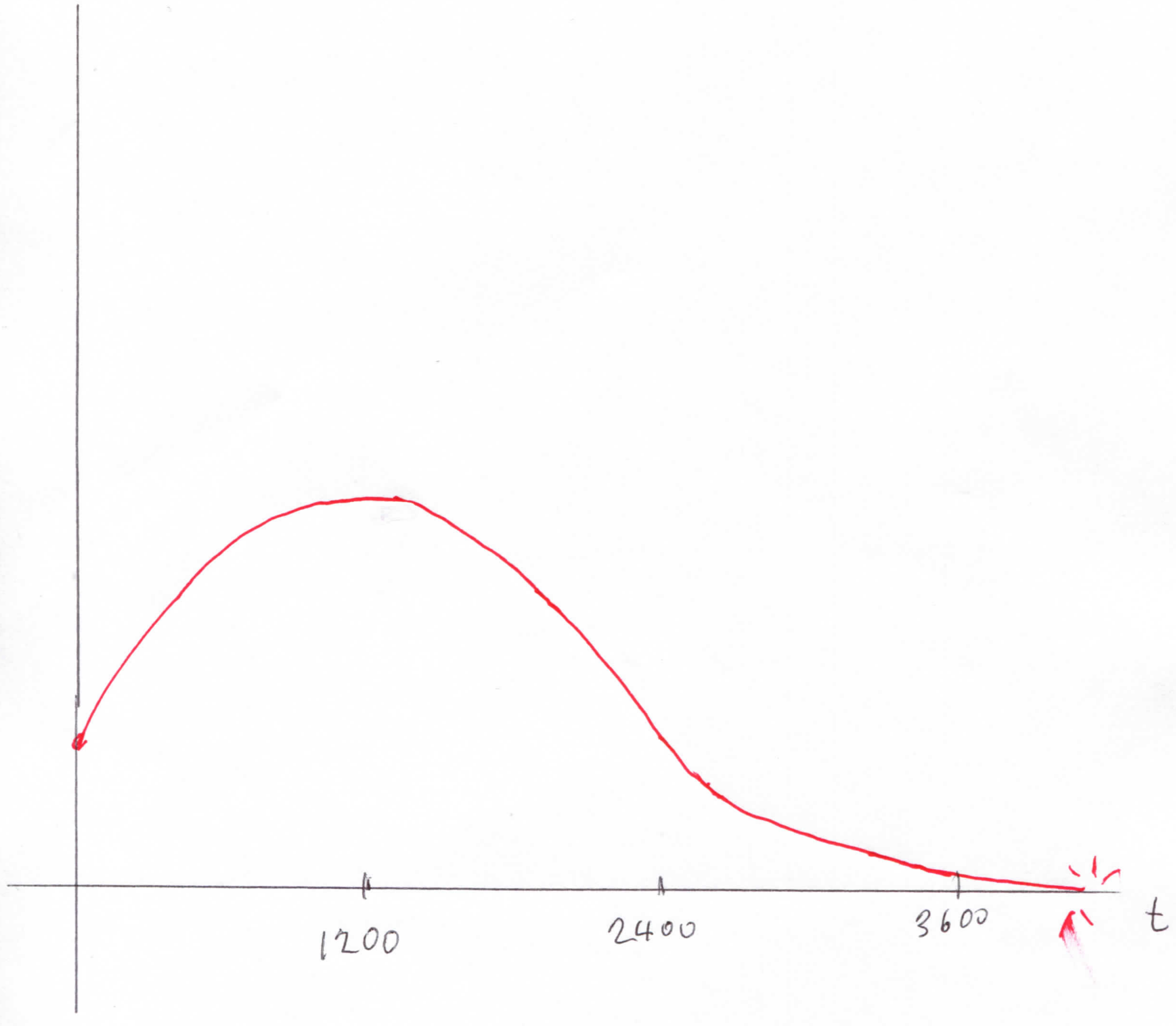
Examples 3.5(a), DRAWING 1



Examples 3.5(a), DRAWING 2



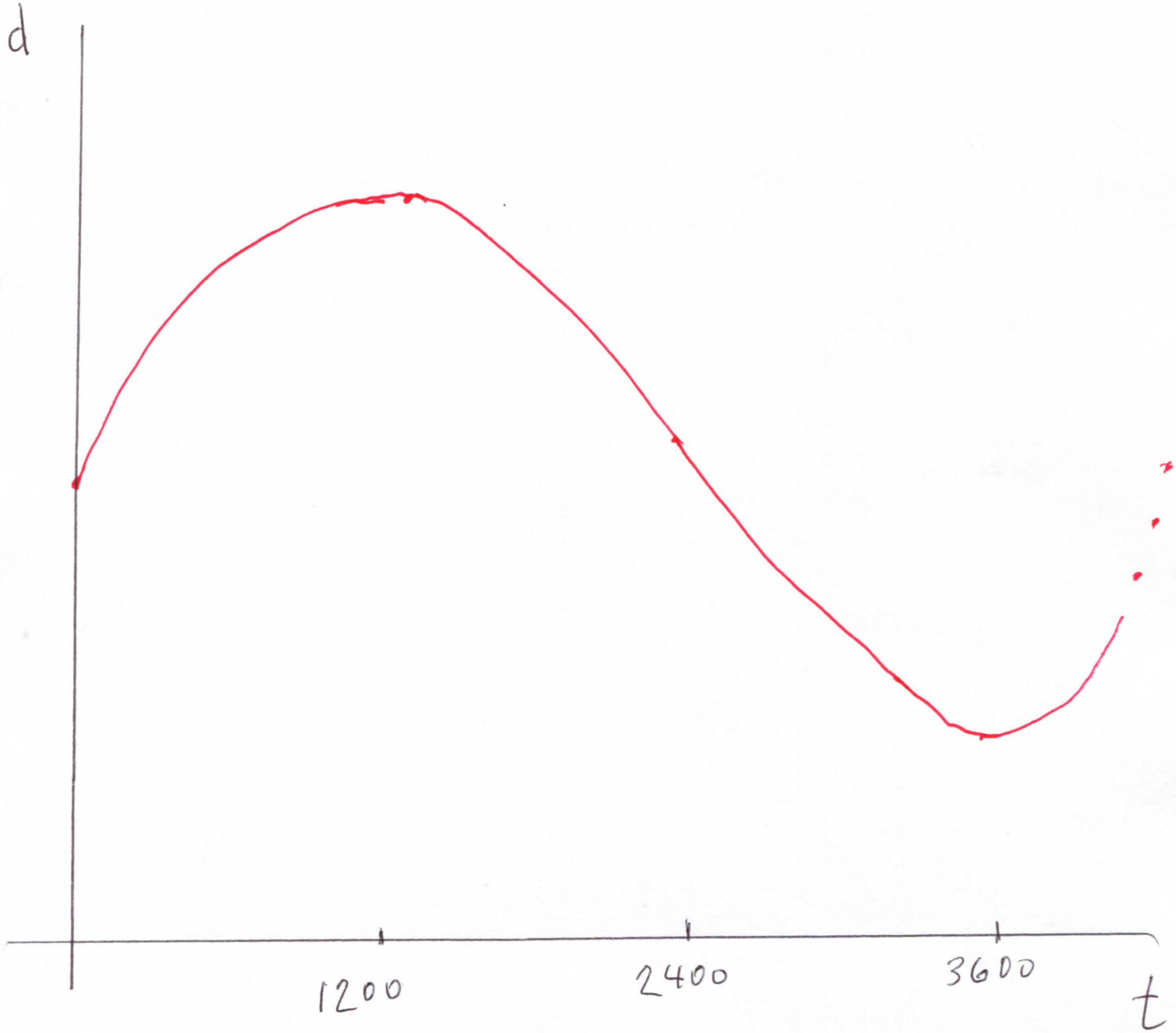
Examples 3.5(a), DRAWING 3



(b) Here's our usual rewriting. We could also paste $v(t) > 0, a(t) > 0$, for $t > 3600$, onto the graph of Examples 2.1(a).

$$0 < t < 1200 : v(t) > 0, a(t) < 0; \quad 1200 < t < 2400 : v(t) < 0, a(t) < 0;$$

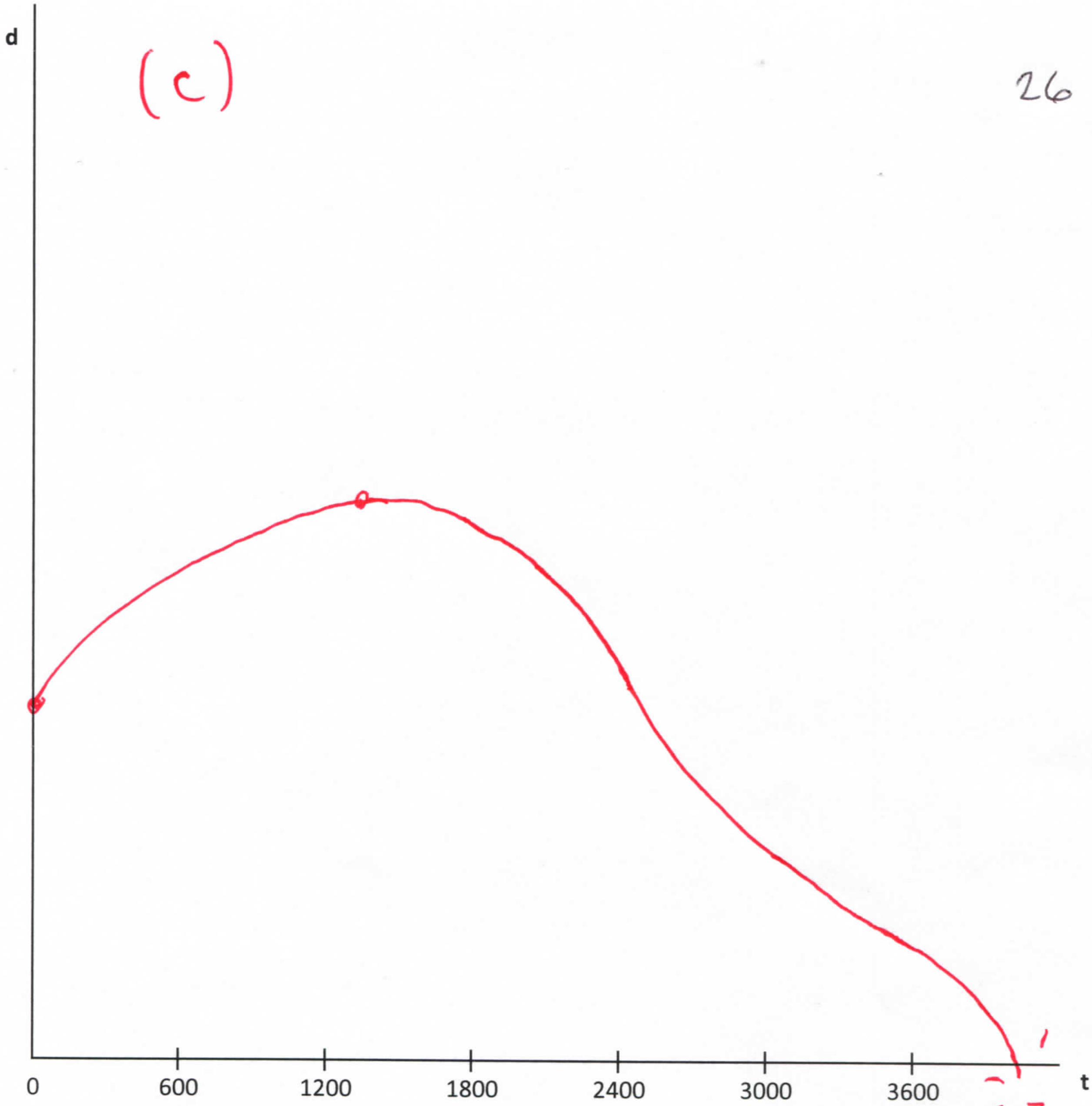
$$2400 < t < 3600 : \quad v(t) < 0, a(t) > 0; \quad t > 3600 : \quad v(t) > 0, a(t) > 0.$$



d gets arbitrarily large

(c)

26



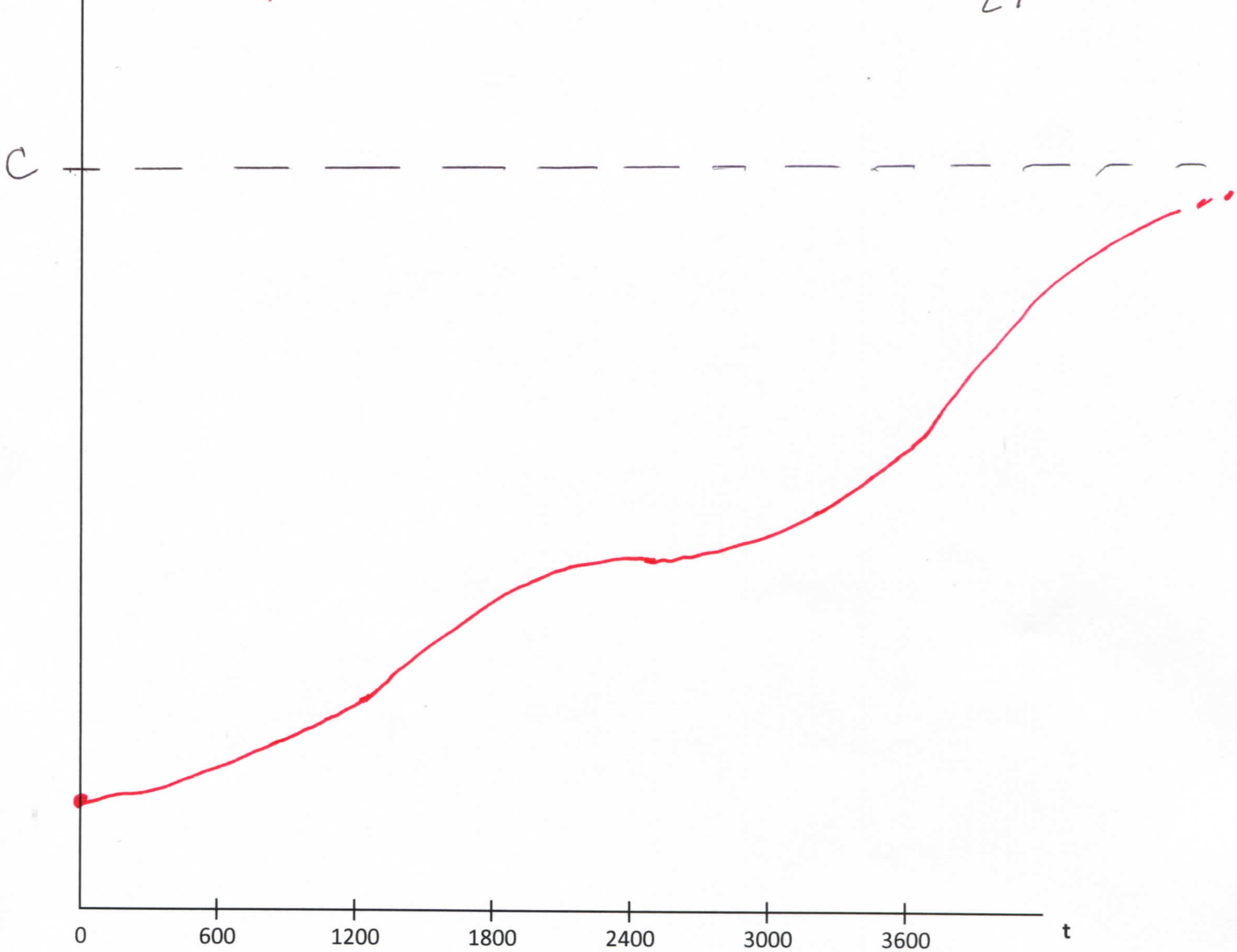
$v>0$	$v>0$	$v<0$	$v<0$	$v<0$	$v<0$	$v<0$
$a<0$	$a<0$	$a<0$	$a<0$	$a>0$	$a>0$	$a<0$

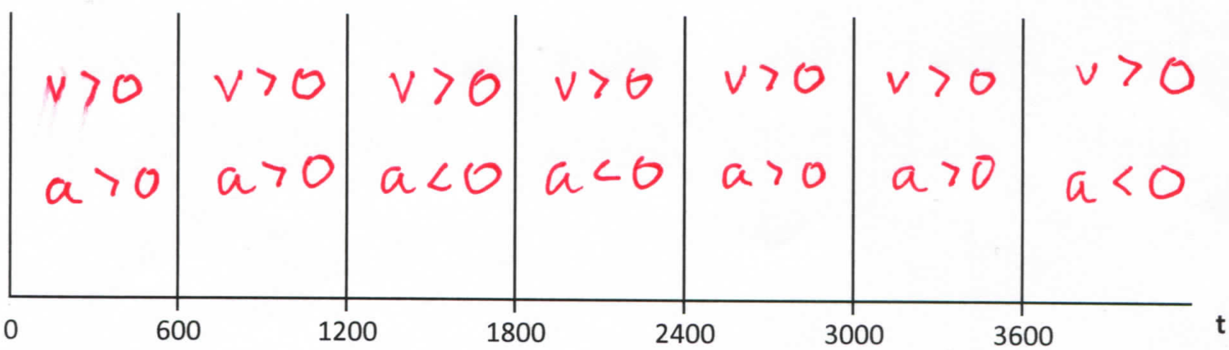
0 600 1200 1800 2400 3000 3600 t

We will hit the sun.

(d) DRAWING 1

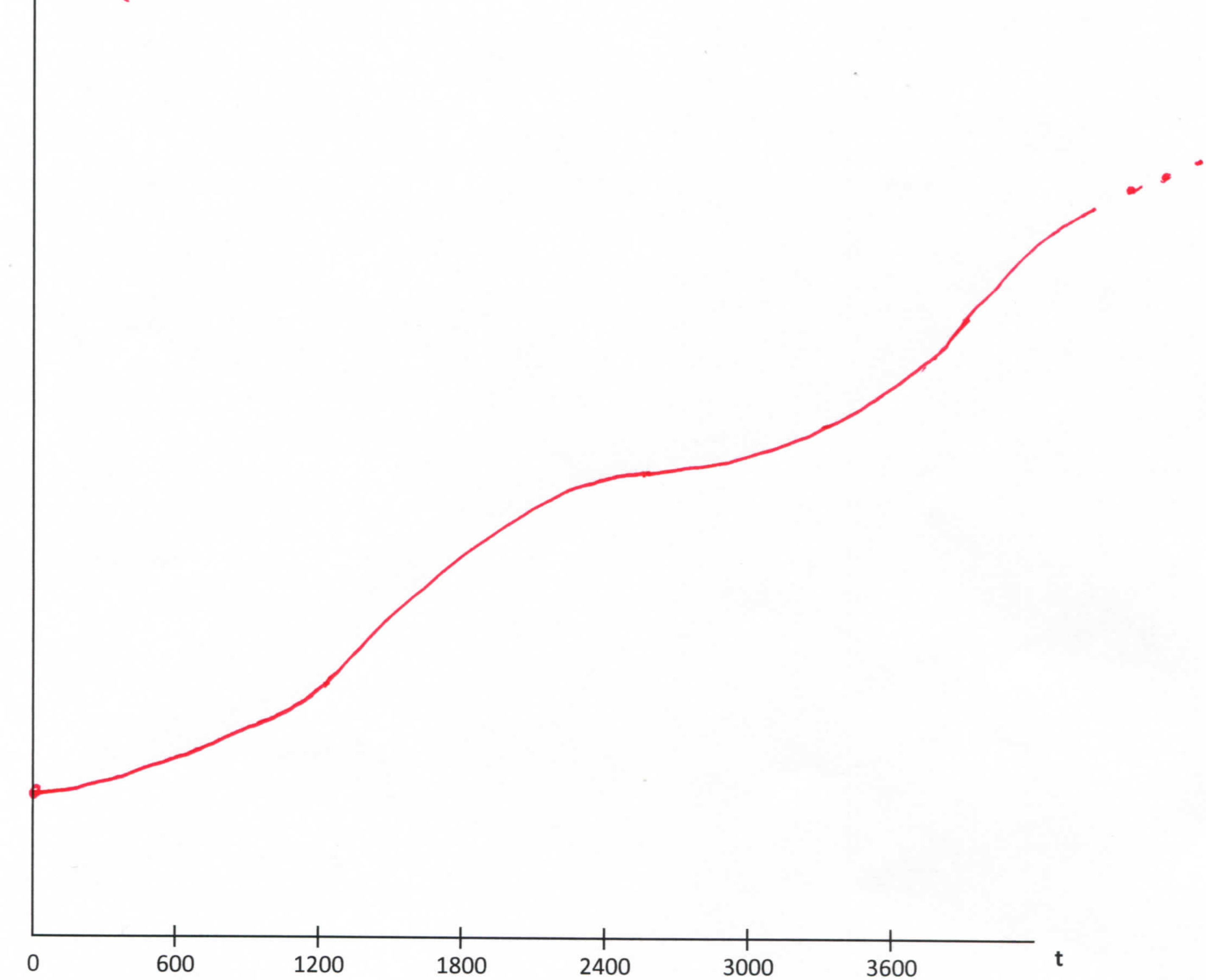
27





horizontal asymptote

(d) DRAWING 2



0–600	600–1200	1200–1800	1800–2400	2400–3000	3000–3600	3600–
$v > 0$	$v > 0$	$v > 0$	$v > 0$	$v > 0$	$v > 0$	$v > 0$
$a > 0$	$a > 0$	$a < 0$	$a < 0$	$a > 0$	$a > 0$	$a < 0$

d gets arbitrarily large

Examples 3.5 nonGRAPH SOLUTIONS.

As with Examples 2.1, we will use the graphs we've just drawn for Examples 3.5 graph solutions to answer the non-graph questions for Examples 3.5.

(a) We might (see first and second graphs drawn for 3.5(a) graph solutions) or might not (see third graph drawn for 3.5(a) graph solutions) have a horizontal asymptote.

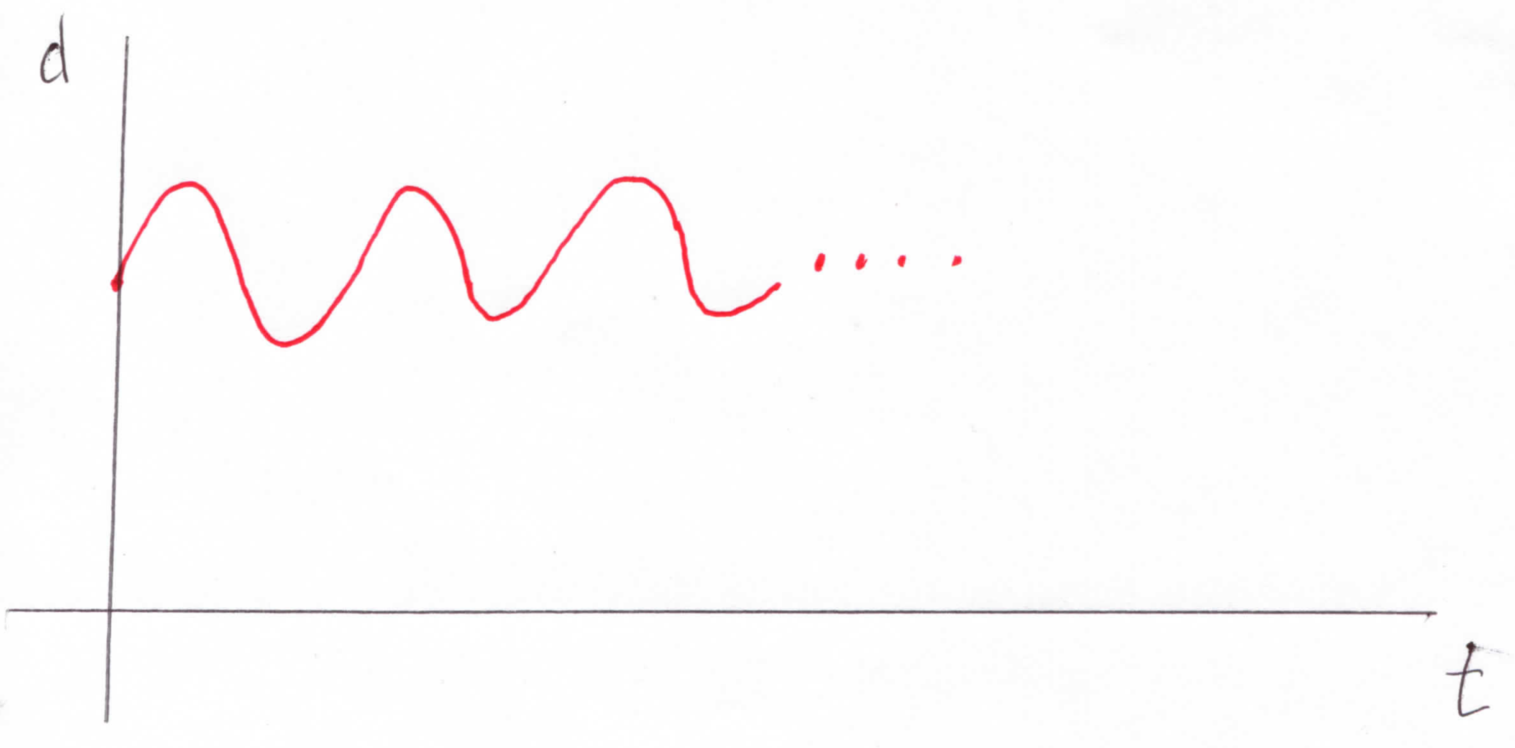
Our spaceship is farthest from the sun 1200 seconds (20 minutes) after leaving the earth. Our spaceship is closest to the sun when we left the earth if we have a horizontal asymptote $d = c$ with c greater than or equal to the distance from the earth to the sun (see first graph drawn for 3.5(a) graph solutions). If we have a horizontal asymptote $d = c$ with c less than the distance from the earth to the sun (see second graph drawn for 3.5(a) graph solutions), then there is no time when we are closest to the earth, since $d(t)$ would get arbitrarily close to c, as t gets large, but would always be greater than c. It is also possible that we will hit the sun (see third graph drawn for 3.5(a) graph solutions).

(b) We will get arbitrarily far away from the sun, by letting sufficient time pass. Our spaceship is closest to the sun either when we left the earth or 3600 seconds (1 hour) after leaving the earth.

(c) Our spaceship is farthest from the sun 1200 seconds (20 minutes) after leaving the earth. We will hit the sun eventually.

(d) We will be closest to the sun when we leave the earth. We might have a horizontal asymptote $d = c$ (see first graph drawn for 3.5(d) graph solutions); there would then be no time when we are farthest from the sun, since $d(t)$ would get arbitrarily close to c, as t gets large, but would always be less than c. It is also possible that we will get arbitrarily far away from the sun, by letting sufficient time pass (see second graph drawn for 3.5(d) graph solutions).

Remarks 3.6. There are other long-term behaviors of interest besides 3.1–3.4. The graph below repeats itself every 2π seconds. Readers who have seen trigonometry might recognize it as $d(t)$ equal to $\sin t$ plus a constant. Functions that repeat themselves define *waves*.



HOMEWORK

In each part, draw all variations, relevant to the questions that follow, of graphs satisfying the specified conditions. To the extent possible, state when our spaceship is closest to the sun or farthest away from the sun; this might include visualizing vertical stretching or compressing, as in Examples 1.4. If we hit the sun, this should be mentioned. If we get arbitrarily far away from the sun, by letting sufficient time pass, this should be mentioned. Any possible asymptotes should be mentioned and drawn with a dotted line.

BLANK GRAPH AVAILABLE for the reader's use at the end of the Magnification

1. $v(t) > 0$ when $0 < t < 600$ or $1800 < t < 2400$ or $2400 < t < 3600$.

$v(t) < 0$ when $600 < t < 1200$ or $1200 < t < 1800$.

$a(t) > 0$ when $1200 < t < 1800$ or $1800 < t < 2400$.

$a(t) < 0$ when $0 < t < 600$ or $600 < t < 1200$ or $2400 < t < 3600$.

Assume no sun crashing.

2. $v(t) > 0$ when $0 < t < 1200$ or $1200 < t < 2400$.

$v(t) < 0$ when $2400 < t < 3600$.

$a(t) > 0$ when $0 < t < 1200$.

$a(t) < 0$ when $1200 < t < 2400$ or $2400 < t < 3600$.

Assume no sun crashing.

3. $v(t) > 0$ when $1200 < t < 1800$ or $1800 < t < 2400$ or $t > 3600$.

$v(t) < 0$ when $0 < t < 600$ or $600 < t < 1200$ or $2400 < t < 3000$ or $3000 < t < 3600$.

$a(t) > 0$ when $600 < t < 1200$ or $1200 < t < 1800$ or $3000 < t < 3600$ or $t > 3600$.

$a(t) < 0$ when $0 < t < 600$ or $1800 < t < 2400$ or $2400 < t < 3000$.

Assume no sun crashing when $t \leq 3600$.

4. $v(t) > 0$ when $0 < t < 600$ or $600 < t < 1200$ or $2400 < t < 3000$.

$v(t) < 0$ when $1200 < t < 1800$ or $1800 < t < 2400$.

$a(t) > 0$ when $0 < t < 600$ or $1800 < t < 2400$ or $2400 < t < 3000$.

$a(t) < 0$ when $600 < t < 1200$ or $1200 < t < 1800$.

Assume no sun crashing.

5. $v(t) > 0$ when $600 < t < 1200$ or $1200 < t < 1800$.

$v(t) < 0$ when $0 < t < 600$ or $1800 < t < 2400$ or $2400 < t < 3000$.

$a(t) > 0$ when $0 < t < 600$ or $600 < t < 1200$ or $2400 < t < 3000$.

$a(t) < 0$ when $1200 < t < 1800$ or $1800 < t < 2400$.

Assume no sun crashing.

6. $v(t) > 0$ when $0 < t < 600$ or $1800 < t < 2400$ or $2400 < t < 3000$.

$v(t) < 0$ when $600 < t < 1200$ or $1200 < t < 1800$ or $t > 3000$ (at least until we hit the sun, if we hit the sun).

$a(t) > 0$ when $1200 < t < 1800$ or $1800 < t < 2400$.

$a(t) < 0$ when $0 < t < 600$ or $600 < t < 1200$ or $2400 < t < 3000$ or $t > 3000$ (at least until we hit the sun, if we hit the sun).

Assume no sun crashing when $t \leq 3000$.

7. $v(t) > 0$ when $600 < t < 1200$ or $1200 < t < 1800$ or $1800 < t < 2400$ or $t > 2400$.

$v(t) < 0$ when $0 < t < 600$.

$a(t) > 0$ when $0 < t < 600$ or $600 < t < 1200$ or $1800 < t < 2400$.

$a(t) < 0$ when $1200 < t < 1800$ or $t > 2400$.

Assume no sun crashing when $t \leq 2400$.

8. $v(t) > 0$ when $1200 < t < 1800$ or $1800 < t < 2400$.

$v(t) < 0$ when $0 < t < 600$ or $600 < t < 1200$ or $2400 < t < 3000$ or $t > 3000$ (at least until we hit the sun, if we hit the sun).

$a(t) > 0$ when $600 < t < 1200$ or $1200 < t < 1800$ or $t > 3000$ (at least until we hit the sun, if we hit the sun).

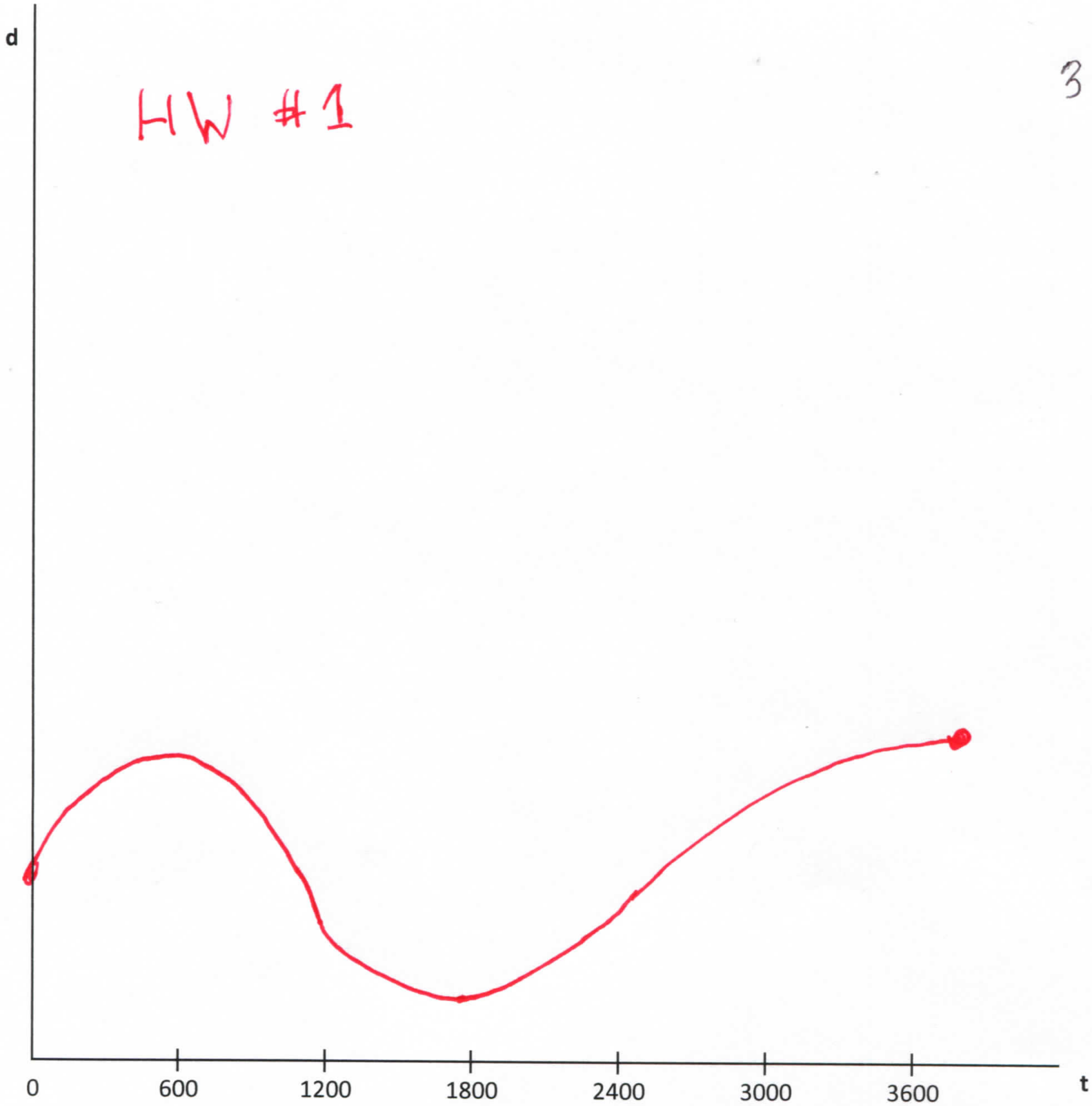
$a(t) < 0$ when $0 < t < 600$ or $1800 < t < 2400$ or $2400 < t < 3000$.

Assume no sun crashing when $t \leq 3000$.

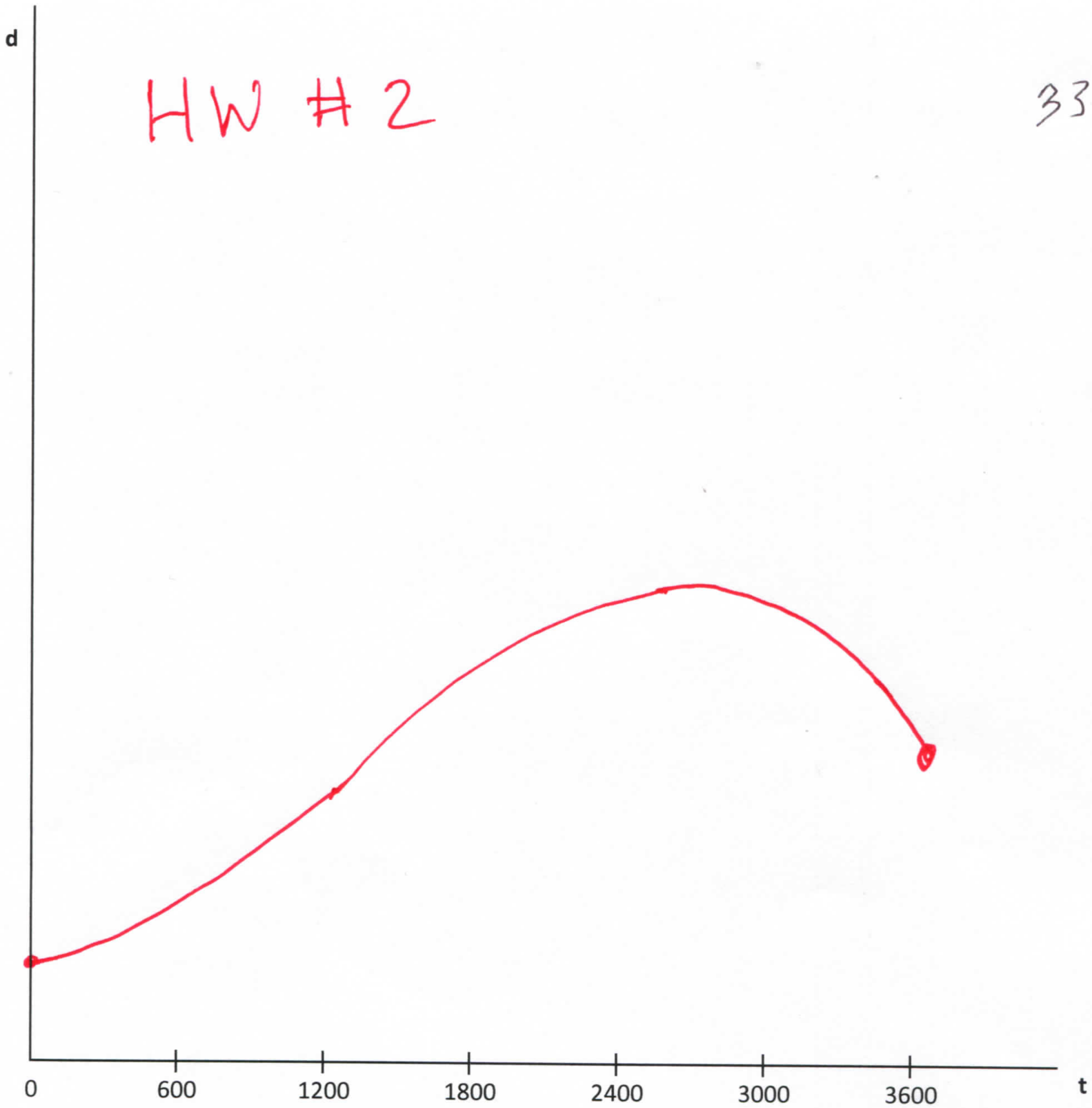
HOMEWORK GRAPH ANSWERS begin on next page

HW #1

32



0–600	600–1200	1200–1800	1800–2400	2400–3000	3000–3600
v > 0	v < 0	v < 0	v > 0	v > 0	v > 0
a < 0	a < 0	a > 0	a > 0	a < 0	a < 0

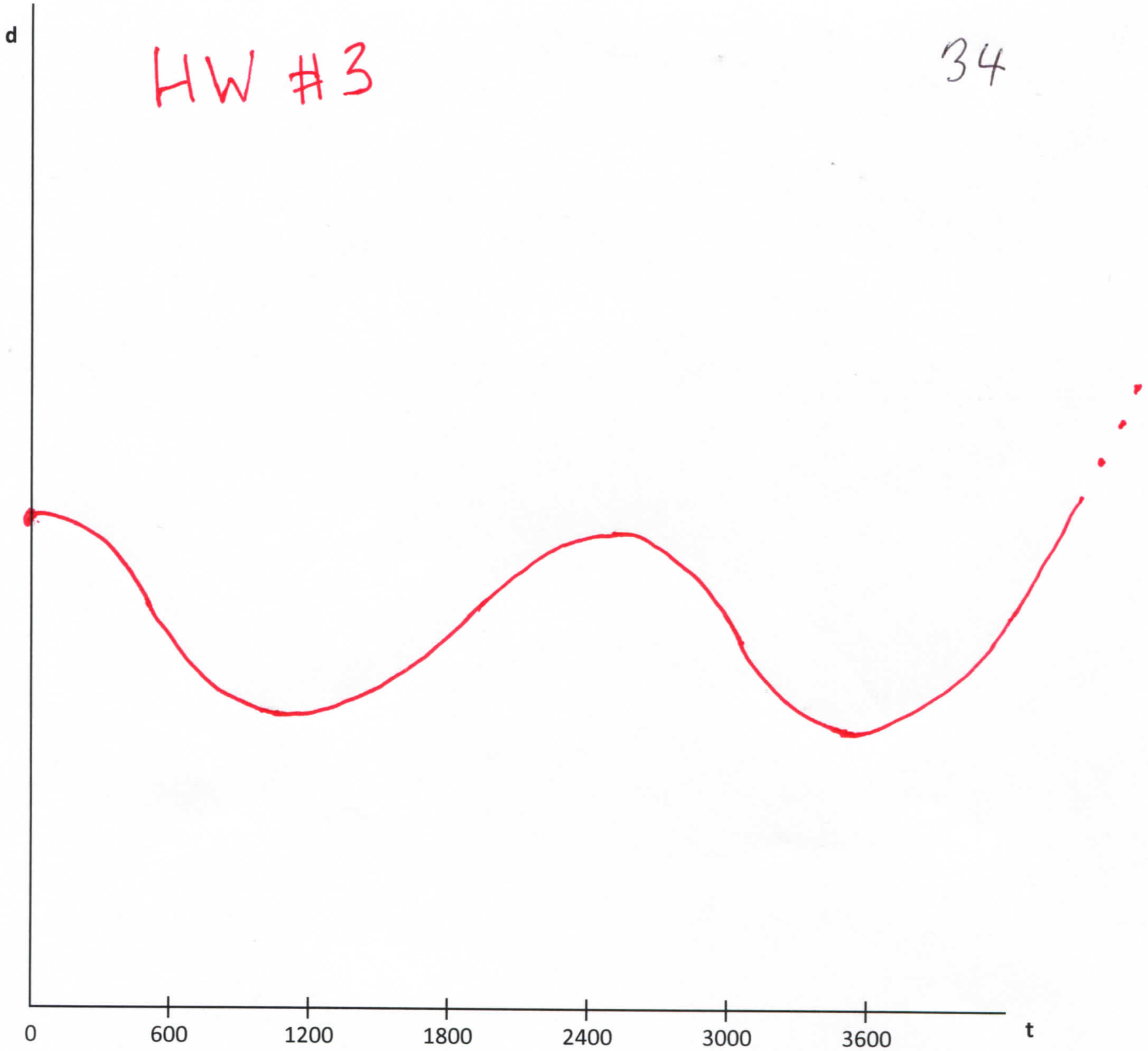


0–600	600–1200	1200–1800	1800–2400	2400–3000	3000–3600
$v>0$	$v>0$	$v>0$	$v>0$	$v<0$	$v<0$
$a>0$	$a>0$	$a<0$	$a<0$	$a<0$	$a<0$

0 600 1200 1800 2400 3000 3600 t

HW #3

34

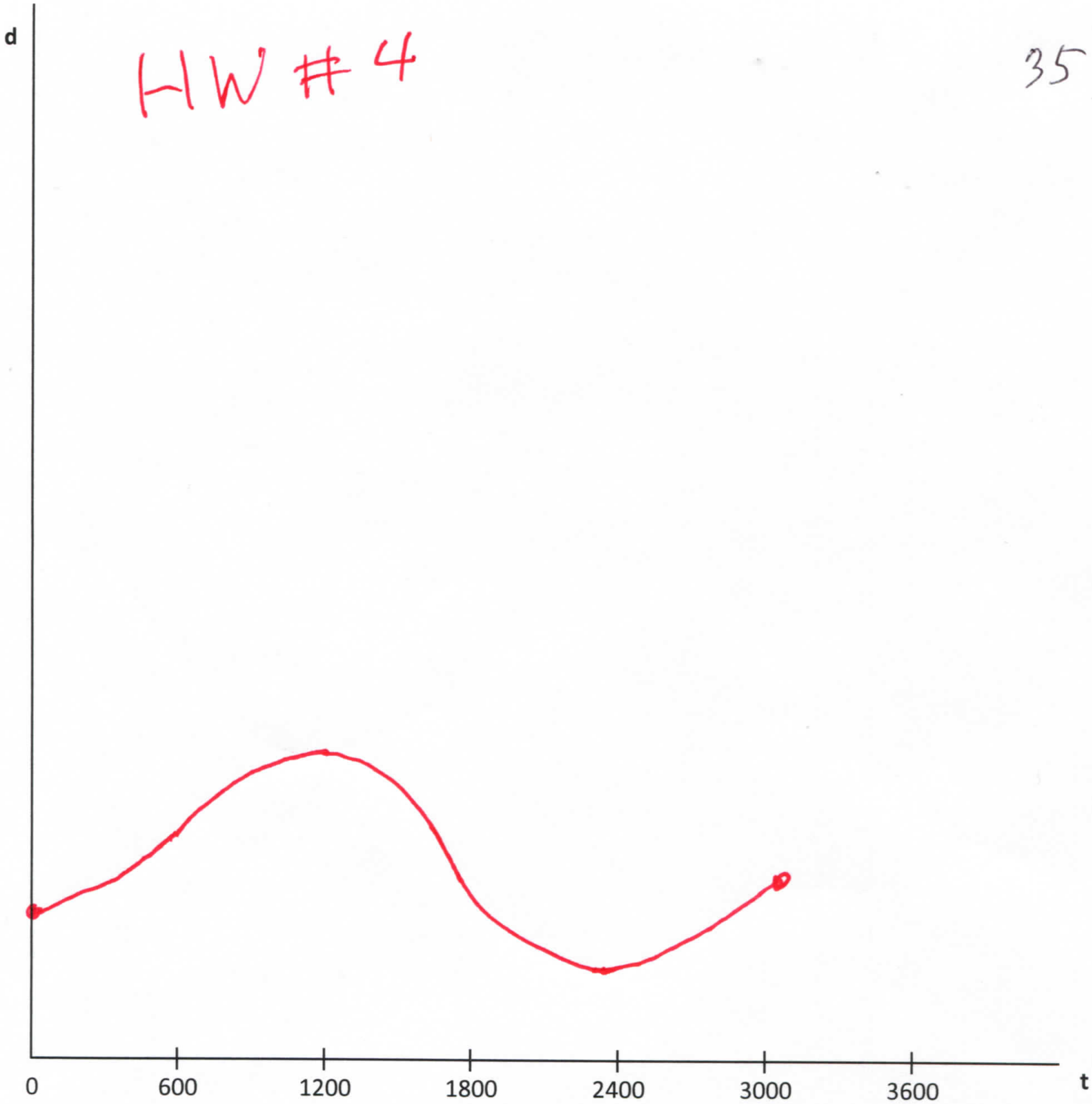


0–600	600–1200	1200–1800	1800–2400	2400–3000	3000–3600	3600–
$v<0$	$v<0$	$v>0$	$v>0$	$v<0$	$v<0$	$v>0$
$a<0$	$a>0$	$a>0$	$a<0$	$a<0$	$a>0$	$a>0$

0 600 1200 1800 2400 3000 3600 t

HW #4

35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0
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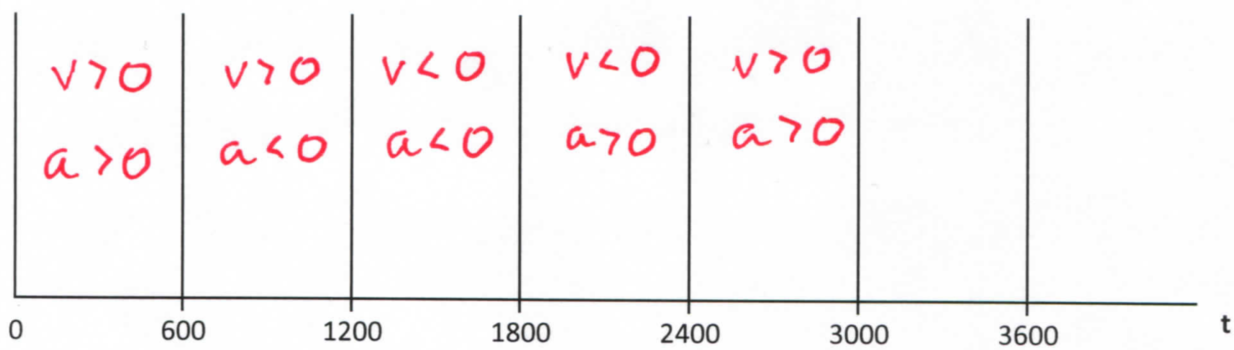
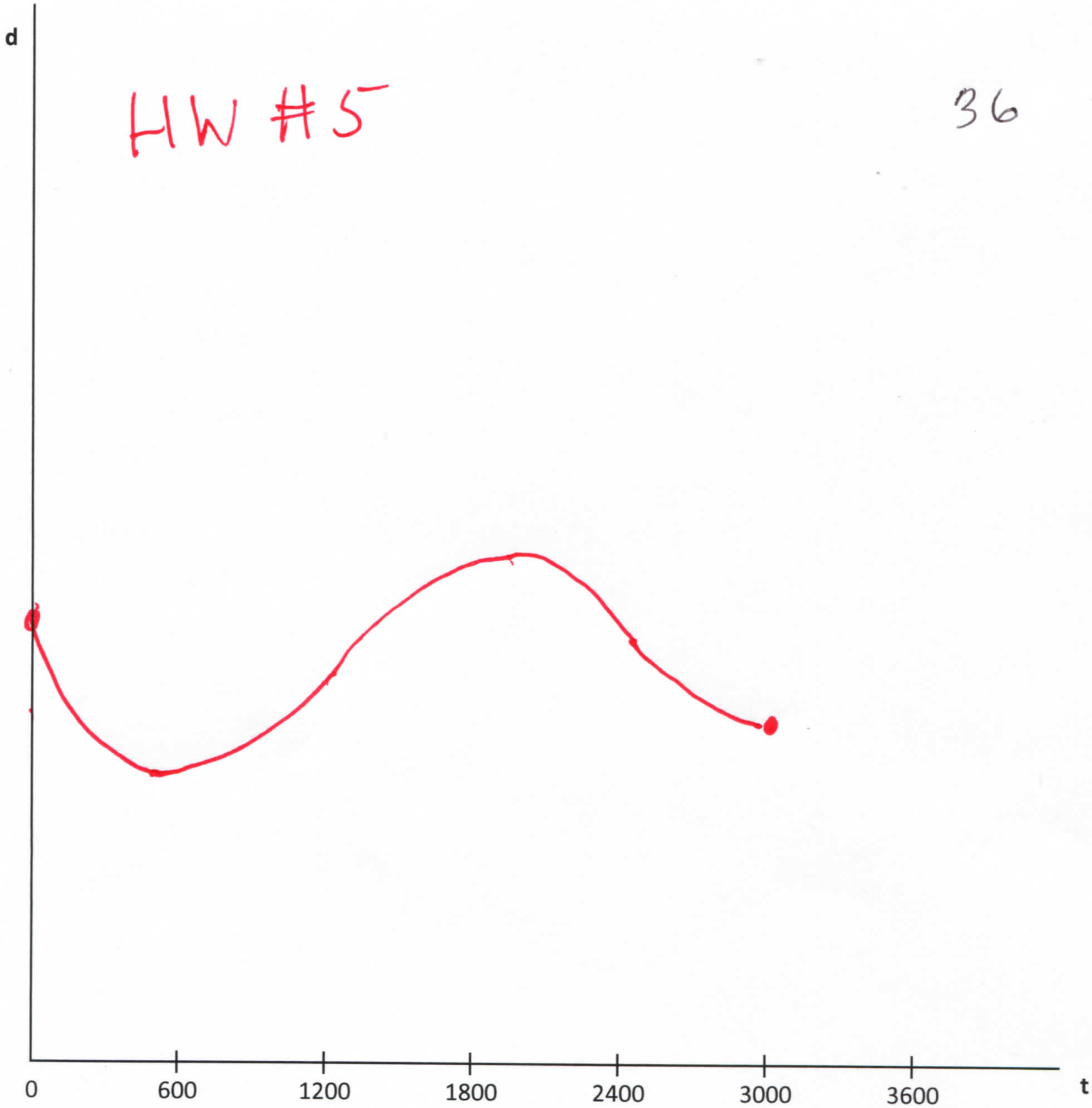
1200
1800
2400
3000
3600


v > 0
a > 0
v > 0
a < 0
v < 0
a < 0
v < 0
a > 0
v > 0
a > 0
0
600
1200
1800
2400
3000
3600
t

HW #5

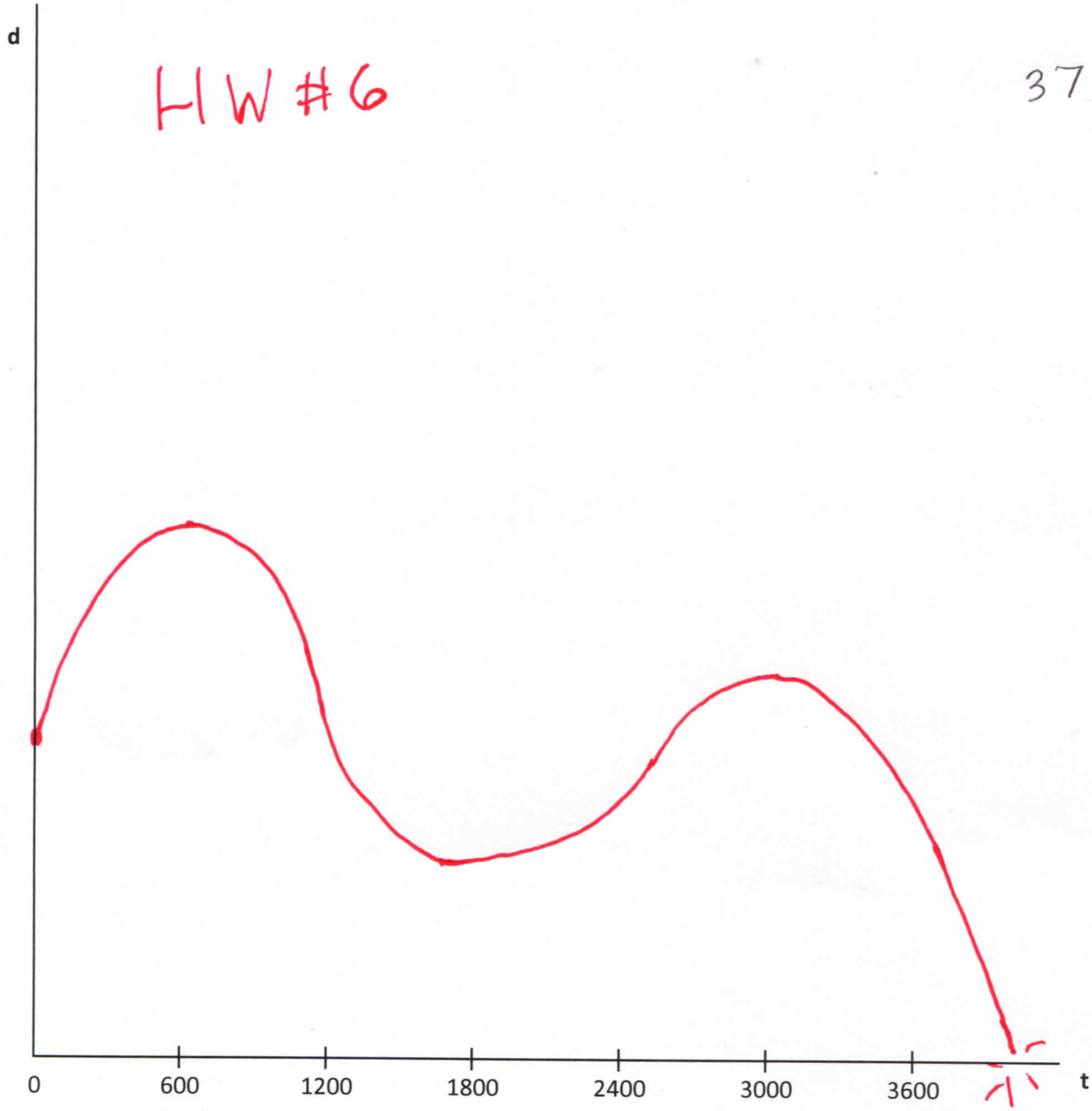
36



0–600	600–1200	1200–1800	1800–2400	2400–3000	3000–3600
$v < 0$	$v > 0$	$v > 0$	$v < 0$	$v < 0$	
$a > 0$	$a > 0$	$a < 0$	$a < 0$	$a > 0$	

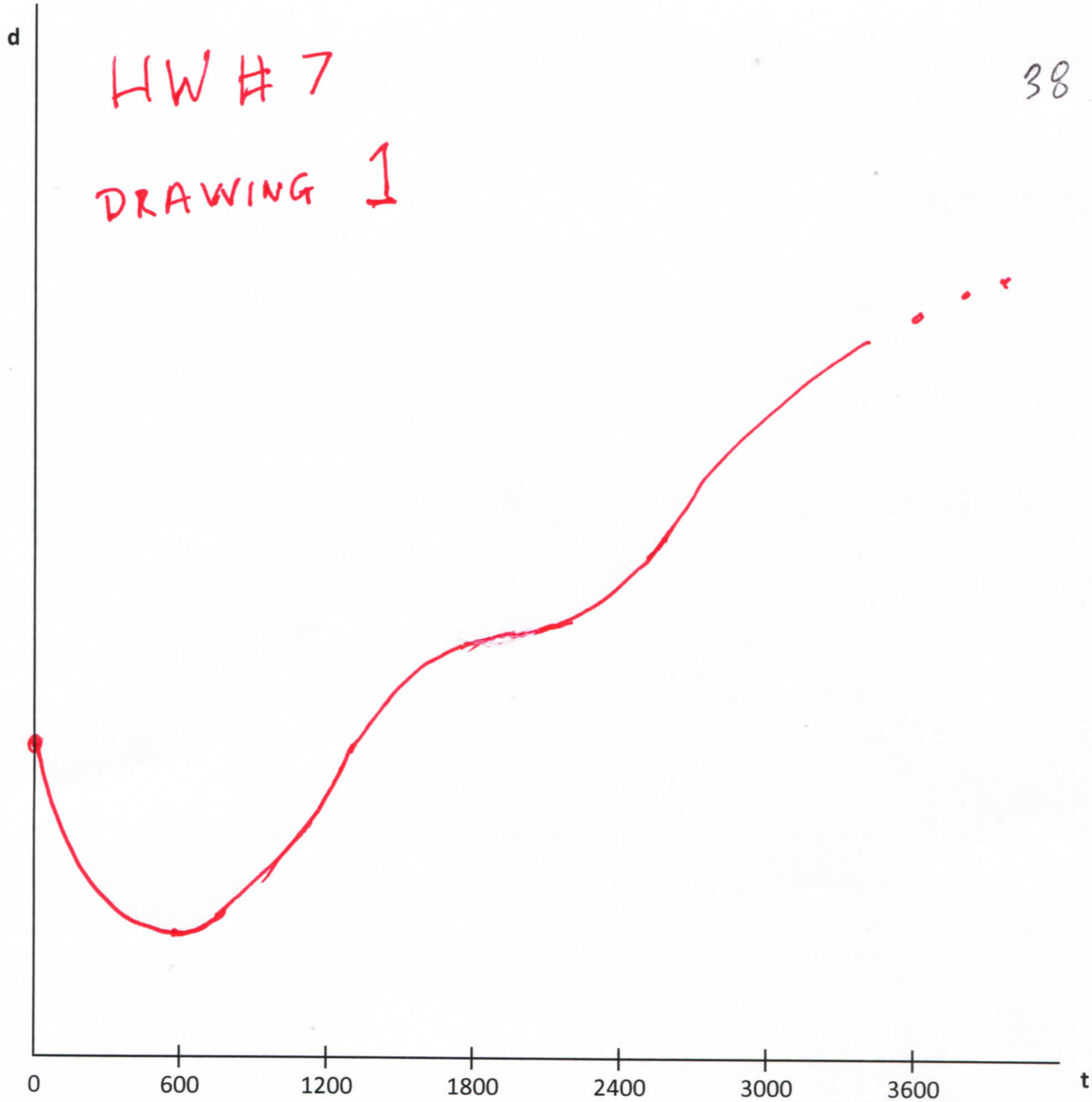
t

HW #6



0–600	600–1200	1200–1800	1800–2400	2400–3000	3000–3600	3600–
v > 0	v < 0	v < 0	v > 0	v > 0	v < 0	v < 0
a < 0	a < 0	a > 0	a > 0	a < 0	a < 0	a < 0

0 600 1200 1800 2400 3000 3600 t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HW #7
DRAWING 1
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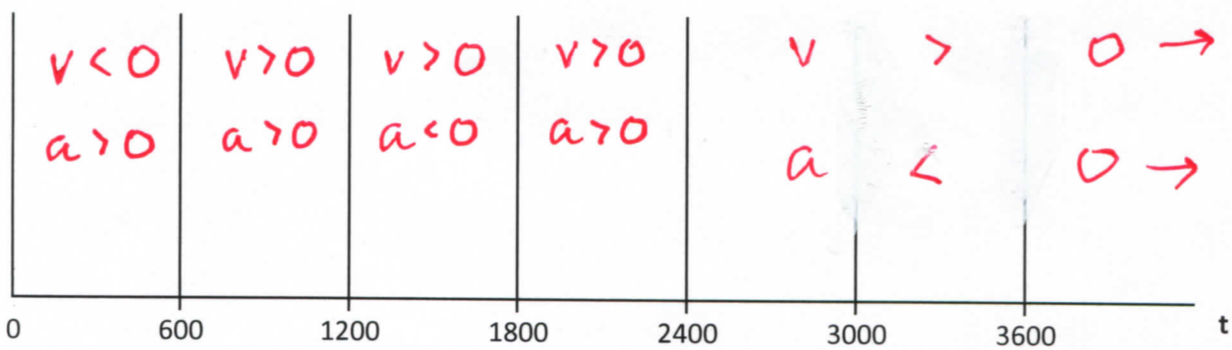
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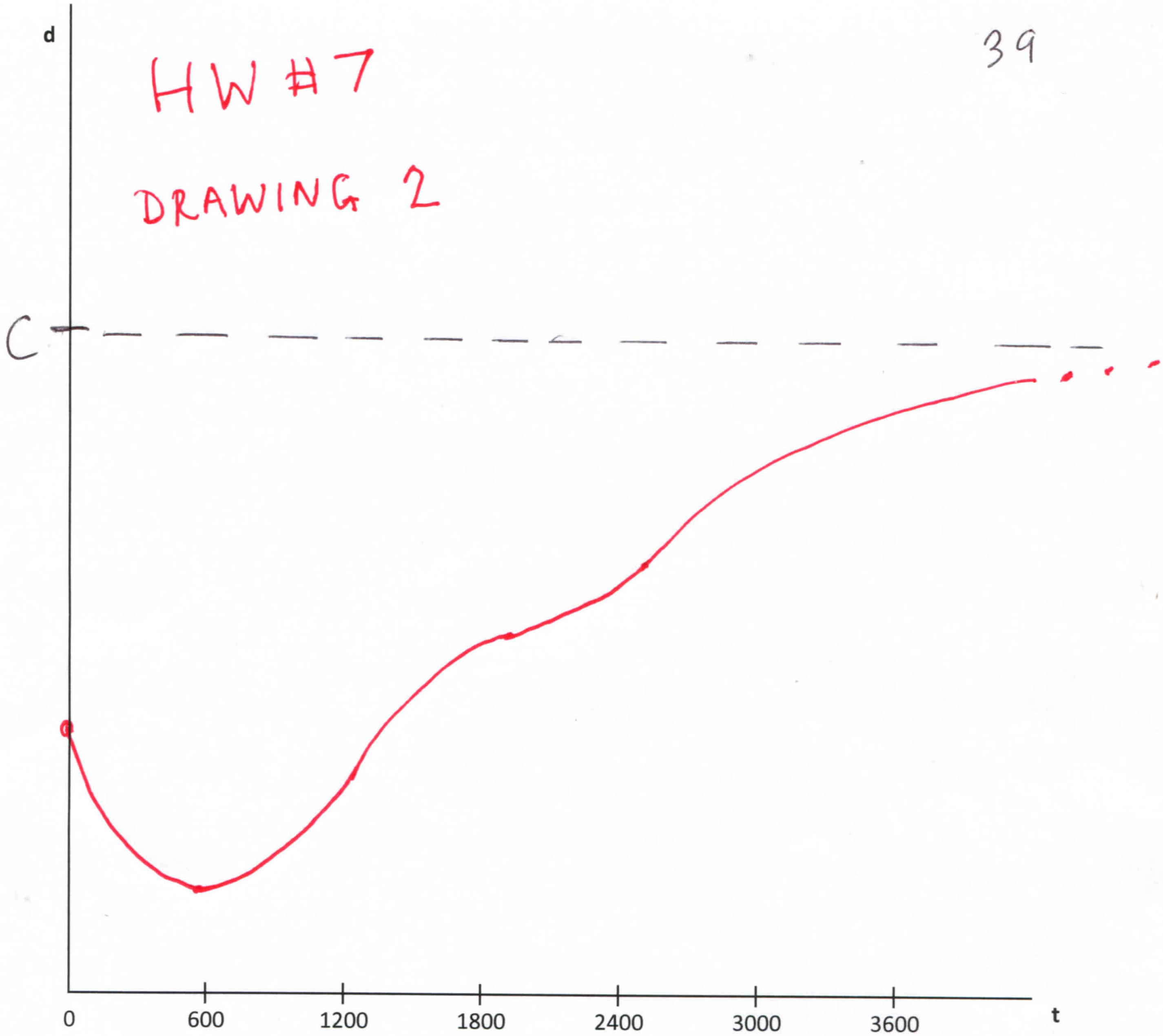
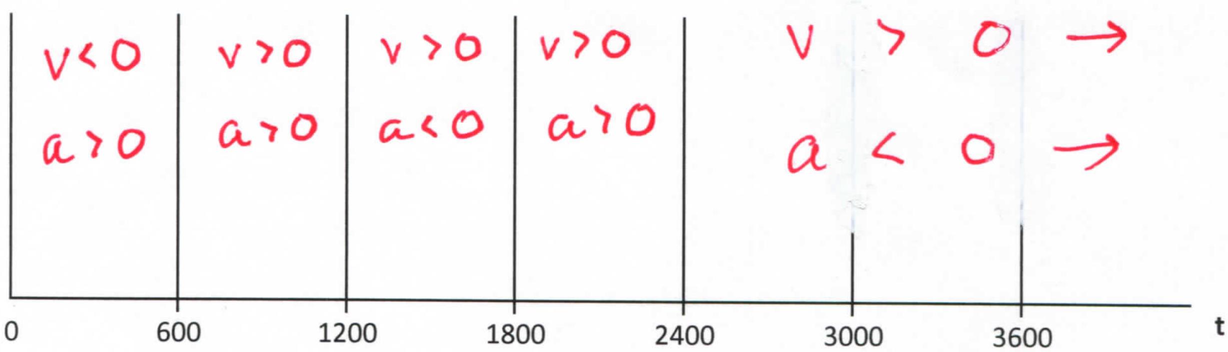
39

HW #7

DRAWING 2

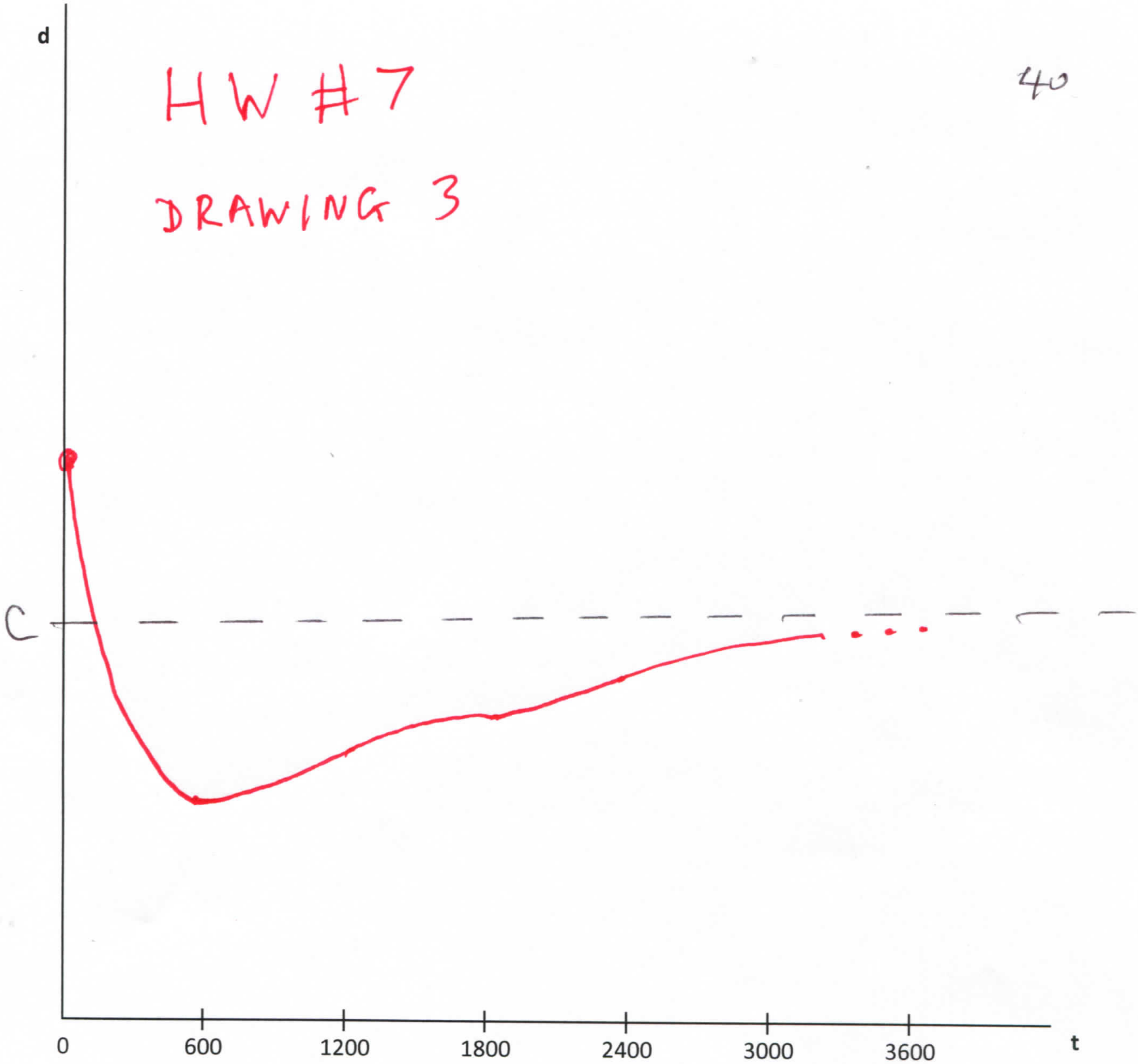


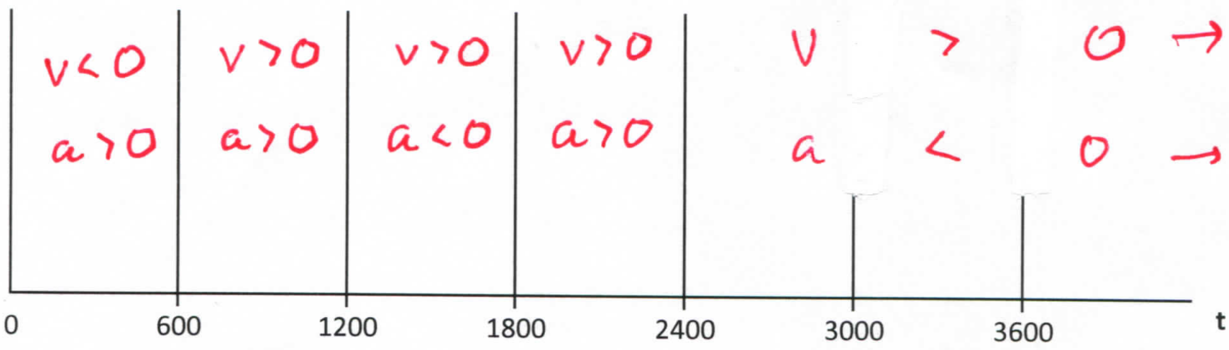


HW #7

DRAWING 3

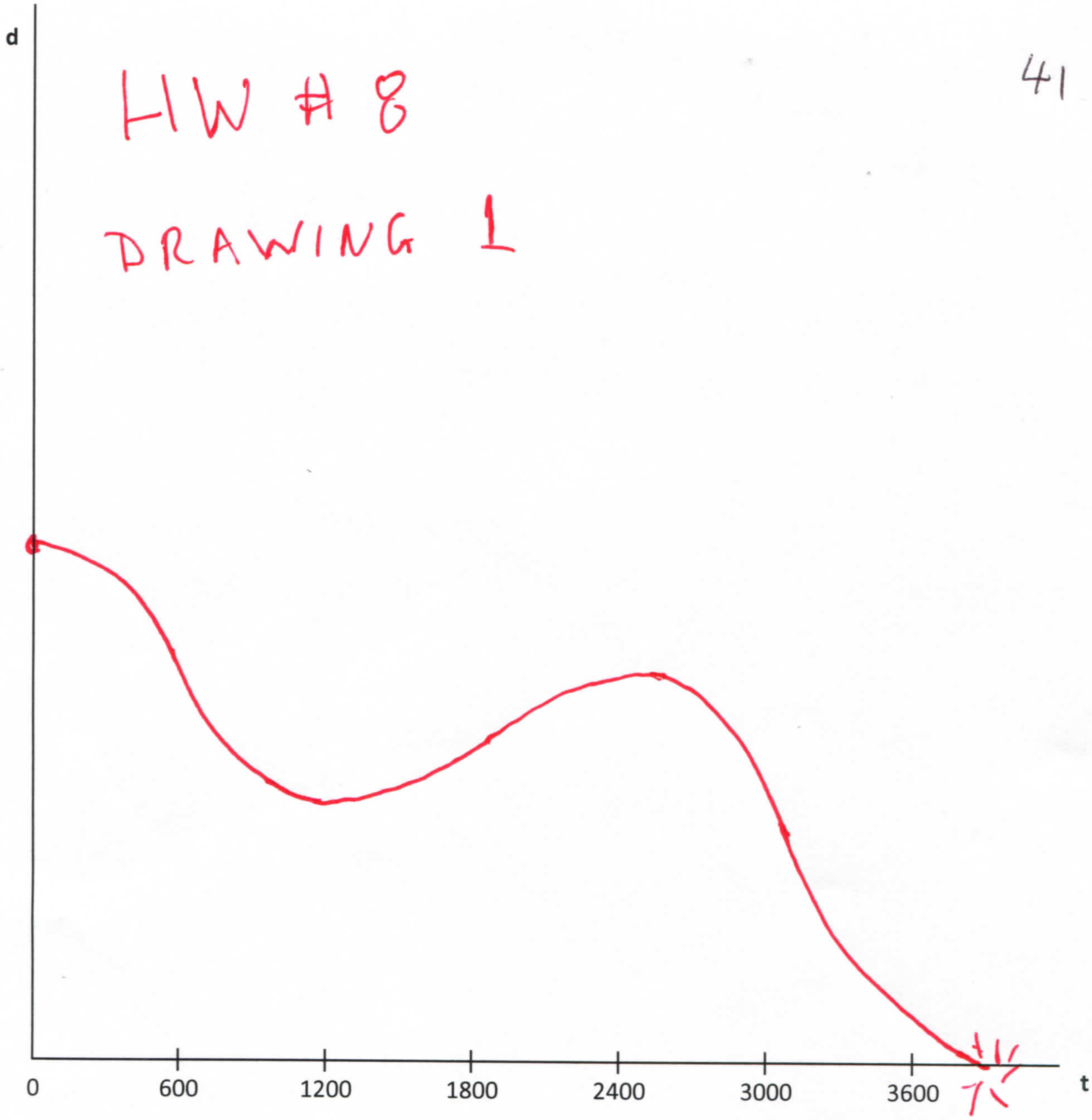
40





HW # 8

DRAWING 1

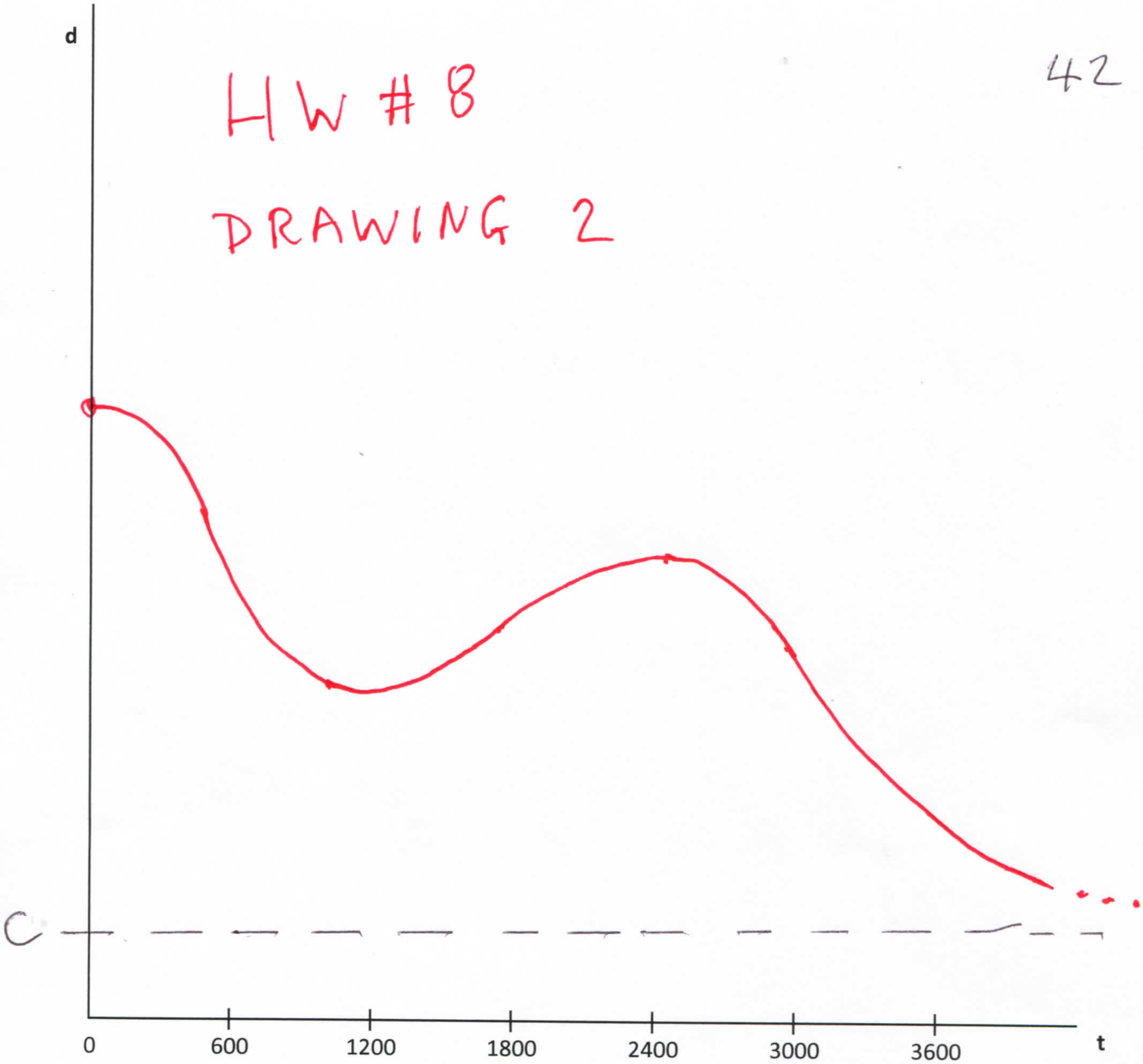


0–600	600–1200	1200–1800	1800–2400	2400–3000	3000–
v<0	v<0	v>0	v>0	v<0	v < 0
a<0	a>0	a>0	a<0	a<0	a > 0

HW #8

DRAWING 2

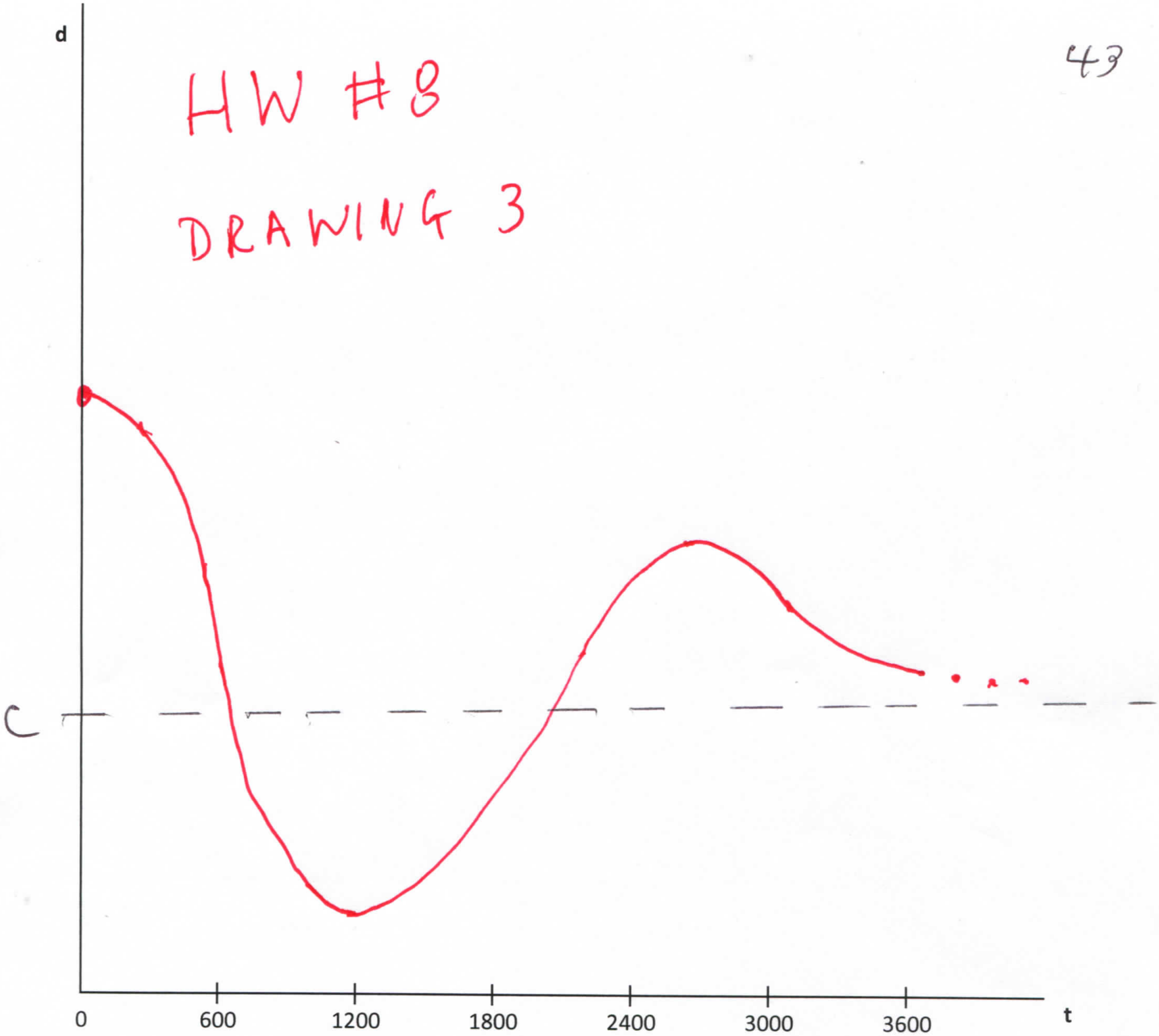
42



t	0–600	600–1200	1200–1800	1800–2400	2400–3000	3000–
v	$v<0$	$v<0$	$v>0$	$v>0$	$v<0$	$v<0$
a	$a<0$	$a>0$	$a>0$	$a<0$	$a<0$	$a>0$

HW #8

DRAWING 3



0–600	600–1200	1200–1800	1800–2400	2400–3000	3000–
v<0	v<0	v>0	v>0	v<0	v < 0
a<0	a>0	a>0	a<0	a<0	a > 0

HOMEWORK nonGRAPH ANSWERS

In each problem, we will use our graph or graphs to answers questions about distance to the sun or asymptotes.

1. Our spaceship is farthest from the sun 600 seconds (10 minutes) after leaving the earth or 3600 seconds (1 hour) after leaving the earth and our spaceship is closest to the sun when we left the earth or 1800 seconds (30 minutes) after leaving the earth.

2. Our spaceship is farthest from the sun 2400 seconds (40 minutes) after leaving the earth and our spaceship is closest to the sun when we left the earth or 3600 seconds (1 hour) after leaving the earth.

3. Our spaceship will get arbitrarily far away from the sun by letting sufficient time pass and our spaceship is closest to the sun 1200 seconds (20 minutes) after leaving the earth or 3600 seconds (1 hour) after leaving the earth.

4. Our spaceship is farthest from the sun 1200 seconds (20 minutes) after leaving the earth or 3000 seconds (50 minutes) after leaving the earth and our spaceship is closest to the sun when we left the earth or 2400 seconds (40 minutes) after leaving the earth.

5. Our spaceship is farthest from the sun when we leave the earth or 1800 seconds (30 minutes) after leaving the earth and our spaceship is closest to the sun 600 seconds (10 minutes) after leaving the earth or 3000 seconds (50 minutes) after leaving the earth.

6. Our spaceship is farthest from the sun 600 seconds (10 minutes) after leaving the earth or 3000 seconds (50 minutes) after leaving the earth.

It is inevitable that our spaceship will hit the sun sometime after 3000 seconds after leaving the earth.

7. Our spaceship will be closest to the sun 600 seconds (10 minutes) after leaving the earth.

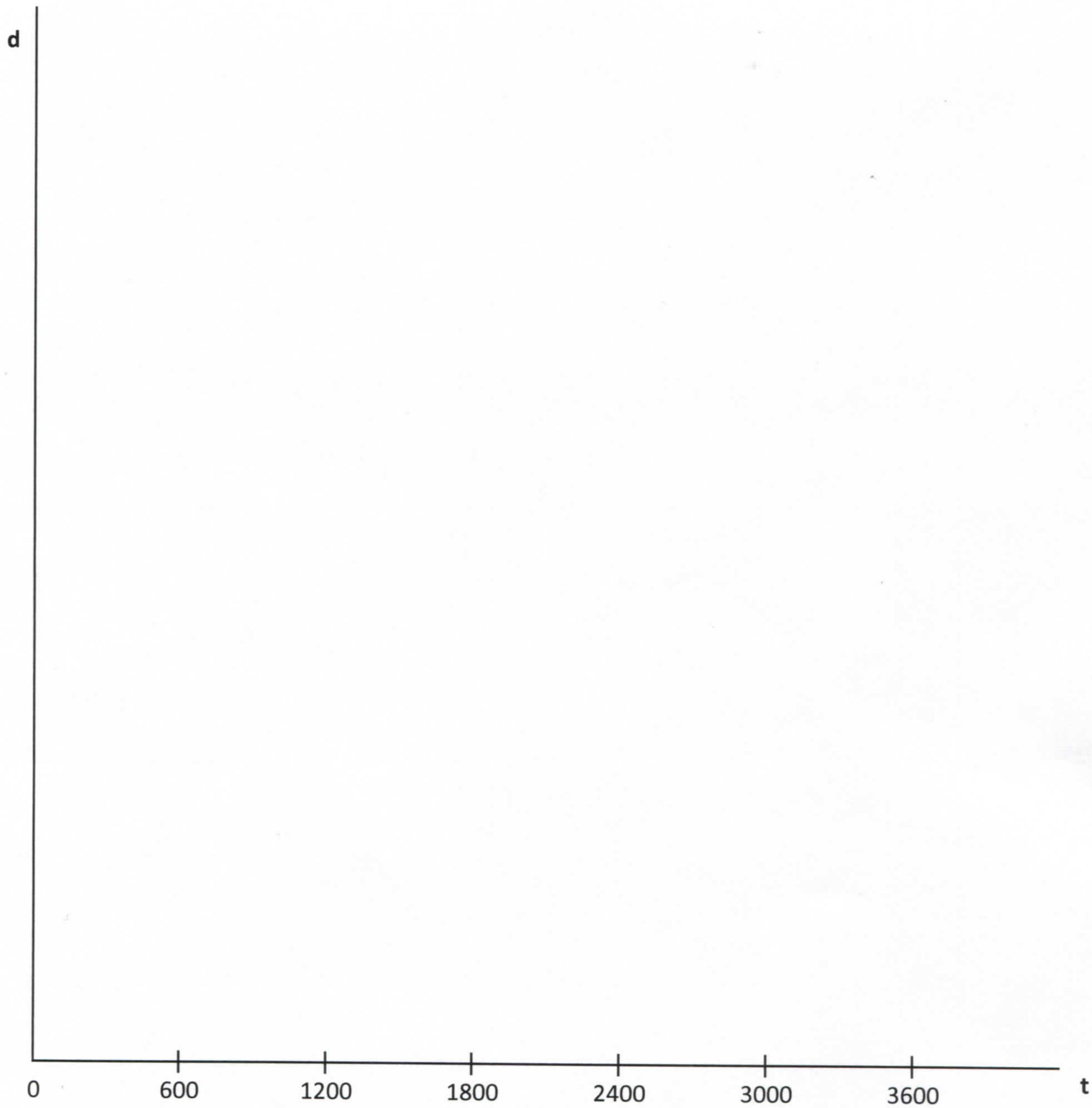
We will either get arbitrarily far away from the sun, by letting sufficient time pass (see DRAWING 1 for HW7 graph solutions) or have a horizontal asymptote $d = c$, for some positive c (see DRAWINGS 2 and 3 for HW7 graph solutions). If, in addition to the latter, $d(0)$, the distance from the earth to the sun, is greater than the asymptotic value c, then our spaceship will be farthest away from the sun when we leave the earth (see DRAWING 3 for HW7 graph solutions).

8. Our spaceship is farthest from the sun either when we leave the earth or 2400 seconds (40 minutes) after leaving the earth. It is possible we will hit the sun sometime after 3000 seconds after leaving the earth (see DRAWING 1 for HW8 graph solutions). If we don't hit the sun, we will have a horizontal asymptote $d = c$, for some nonnegative c (see DRAWINGS 2 and 3 for HW8 graph solutions). If, in addition, $d(1200)$, our distance to the sun 1200 seconds after leaving the earth, is less than the asymptotic value c, then the spaceship will be closest to the sun 1200 seconds (20 minutes) after leaving the earth (see DRAWING 3 for HW8 graph solutions).

REFERENCES

1. John Saxon, "Algebra 1. An Incremental Development," Second Edition, Saxon Publishers, Inc., 1990.

2. John Saxon, "Algebra 2. An Incremental Development," Second Edition, Saxon Publishers, Inc., 1991.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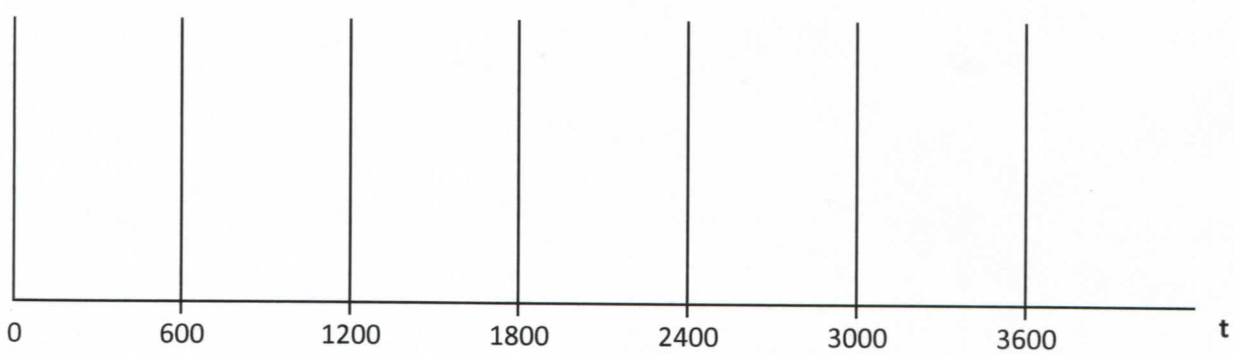
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