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Statistical Inference on Mean and Proportion
MATHematics MAGnification™

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STATISTICAL INFERENCE on MEAN and PROPORTION MAGNIFICATION

This is one of a series of very short books on math, statistics, and physics called "Math Magnifications." The "magnification" refers to focusing on a particular topic that is pivotal in or emblematic of mathematics.

OUTLINE

This magnification constructs confidence intervals and hypothesis tests for mean and proportion, in an organized, unified way, that was derived for a special case in prior magnifications (see [4] for confidence intervals and [5] for hypothesis testing).

As promised in [4] and [5], the constructions of this magnification are essentially the same as what appeared with motivation and detailed derivation in [4] and [5].

Many examples are given. The introduction includes the definition of a *controlled experiment*, central to science.

Prerequisites for this magnification are algebra ([8] is more than sufficient), the basic terminology of statistics, as in [3] or [6], and the definitions and motivation for confidence intervals and hypothesis testing, as in [6] or [4] and [5]. The basic language of probability, as in [2], is also needed for this magnification. A more thorough understanding of probability, as in [1] or [6], would be helpful but is not necessary.

We will adopt, in this Magnification, the custom of stating that numbers from probability tables are *equal* to what we want, although they are usually only an approximation. We also assume the popular custom of upper-case letters, e.g., X, being random variables, with the corresponding lower-case letter, e.g., x, being a numerical measurement of X.

1. INTRODUCTION

Statistics worries about populations too large for us to know much about individuals. The most we can hope for is information about some measurement or characteristic of the population, called a **parameter**. This is where **sampling** begins, as in [3].

Two natural parameters (see [3, Definitions 6] are **mean** or **average** and **proportion** or **percent**. For example, if we were concerned about people's yearly incomes, we could calculate the average yearly income, or we could calculate the proportion of people who make enough money to survive.

Of particular interest is the *difference* between two population means or proportions, for the following reason. If we want to estimate the effect of a treatment, we would apply said treatment to a population X_1, and compare it to another population X_2, that does *not* receive the treatment. Thus we would perform statistical inference on $(\mu_1 - \mu_2)$, a difference of means, or $(p_1 - p_2)$, a difference of proportions. X_2 is called a **control** or **control group**. Ideally, X_2 should differ from X_1 only in not receiving the treatment; statistical inference on differences would then qualify as a **controlled experiment**.

For example, if we wanted to study the effect of vitamin pills on people, one group of people would receive vitamin pills, while another (control) group would be deprived of vitamin pills. After a month, we might compare the proportion of sick people in the vitamin pill-taking group to the proportion of sick people in the control group. Alternatively, we might compare the average number of pounds that the people taking vitamin pills can military press to the average number of pounds the people in the control group can military press. To be a controlled experiment, we would want the two groups of people to be the same, except in their vitamin pill consumption. For example, if the people in the control group were much older, on average, than the people in the vitamin pill-taking group, it would not be a controlled experiment.

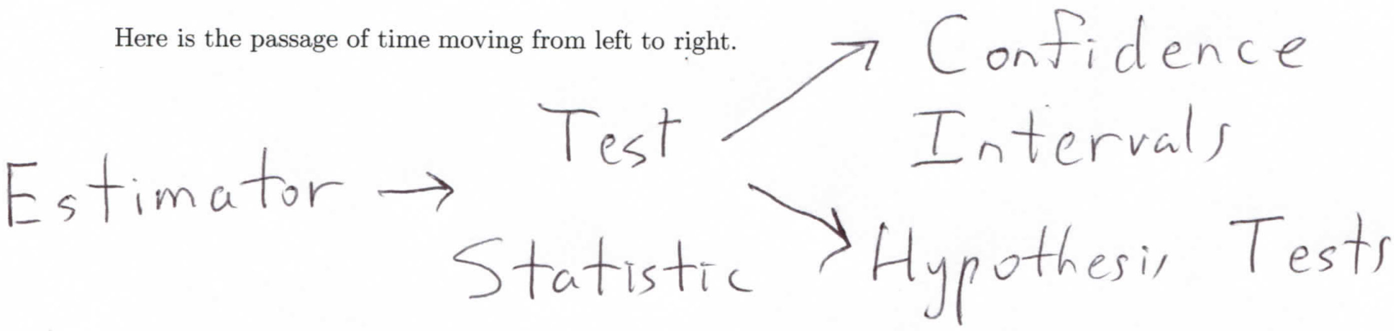
The primary outcomes of statistical inference are confidence intervals and hypothesis tests. We have seen both of these, for the (unknown) population mean μ, of a population that is normally distributed with known standard deviation σ; see [4] and [5]. We have seen, in this scenario (see horizontal line (1) in the tables in Section 2 of this magnification) how both confidence intervals and hypothesis tests are constructed from the *test statistic*

$$\frac{(\overline{X} - \mu)}{\frac{\sigma}{\sqrt{n}}} = Z \quad (*),$$

which is a perturbation of the *sample mean* $\overline{X}$, the preferred *estimator* of μ. Actual numbers for relevant probabilities appear when we haul out tables of Z probabilities, as in the tables at the end of this magnification. See [4, Chapter 2].

Notice the flow chart for statistical inference about a parameter θ, illustrated in the special case of [4] and [5]. Some function of the data, called an *estimator*, that we believe is good for approximating θ, is constructed. Said estimator is then modified to something with a familiar (meaning tables for it exist) probability distribution, called a *test statistic*. The test statistic is used for both confidence intervals and hypothesis testing.

Here is the passage of time moving from left to right.



The parameters of interest in this magnification are mean μ and proportion p; see [3, Definitions 6] for definitions of mean and proportion, of both samples and populations.

Following the flow chart above, Table 2.3 gives test statistics, Table 2.4 gives confidence intervals, and Tables 2.5 and 2.6 give hypothesis tests; Table 2.5 with P-values (see Theorem 2.2), Table 2.6 with rejection regions.

Definitions 1.1. It is often not realistic to assume, as in [4] and [5], that σ is known; we must estimate it from data $x_1, x_2, \ldots, x_n$ with the **sample variance**

$$s^2 \equiv \frac{1}{(n-1)}\sum_{k=1}^{n}(x_k-\overline{x})^2 \equiv \frac{1}{(n-1)}\left[(x_1-\overline{x})^2+(x_2-\overline{x})^2+\cdots+(x_n-\overline{x})^2\right],$$

a popular estimator of the population variance σ^2, or the **sample standard deviation** $s \equiv \sqrt{s^2}$, a popular estimator of σ.

Example 1.2. Data $x_1 = 10, x_2 = -2, x_3 = 5, x_4 = -7, x_5 = 4$ has a sample mean of

$$\frac{1}{5}\left[10+(-2)+5+(-7)+4\right]=2,$$

thus the sample variance is

$$s^2 = \frac{1}{(5-1)}\left[(10-2)^2+(-2-2)^2+(5-2)^2+(-7-2)^2+(4-2)^2\right] = \frac{1}{4}\left[64+16+9+81+4\right] = 43.5$$

and the sample standard deviation is

$$s = \sqrt{43.5} \sim 6.60.$$

Computational Formula 1.3.

$$\sum_k (x_k-\overline{x})^2 = \sum_k x_k^2 - \frac{1}{n}\left(\sum_k x_k\right)^2 \equiv (x_1^2+x_2^2+\cdots+x_n^2) - \frac{1}{n}(x_1+x_2+\cdots+x_n)^2.$$

Example 1.4. For the data in Example 1.2, s^2 equals

$$\frac{1}{(5-1)}\left[(10^2+(-2)^2+5^2+(-7)^2+4^2)-\frac{1}{5}(10+(-2)+5+(-7)+4)^2\right] = \frac{1}{4}\left[194-\frac{1}{5}(10)^2\right] = 43.5.$$

Remarks 1.5. In Definitions 1.1, the dubious reader might ask "Why do we divide by $(n-1)$ instead of n?"

$$\frac{1}{n}\sum_{k=1}^{n}(x_k-\overline{x})^2 \equiv \frac{1}{n}\left[(x_1-\overline{x})^2+(x_2-\overline{x})^2+\cdots+(x_n-\overline{x})^2\right]$$

would be the "average of the squared deviations from the sample mean."

We will answer the dubious reader in two ways.

First, $\{(x_k - \overline{x})\}_{k=1}^n$ is not all ordered n tuples $\{y_k\}_{k=1}^n$, since $\sum_k (x_k - \overline{x})$ can be shown to be zero. The set of n tuples we are dealing with have dimension $(n-1)$, or $(n-1)$ degrees of freedom, since we lost a dimension or degree of freedom by the condition that the coordinates add up to zero.

Second, it can be shown that the expected value of $\frac{1}{(n-1)} \sum_{k=1}^n (X_k - \overline{X})^2$ equals σ^2; this is called being an *unbiased estimator* of σ^2, which is a good thing.

1.6. Large Sample good news about S. Just as the Central Limit Theorem (see [1] or [6]) asserts that the test statistic $\frac{(\overline{X}-\mu)}{\frac{\sigma}{\sqrt{n}}}$ in (*) above is a reasonably good approximation of Z, for arbitrary X, for n sufficiently large ($n > 30$ is traditional), the test statistic $\frac{(\overline{X}-\mu)}{\frac{S}{\sqrt{n}}}$ is a reasonably good approximation of Z, for arbitrary X, for n a little larger than required for the Central Limit Theorem; $n > 40$ is traditional (see horizontal lines (2) and (5) in the tables in Chapter 2 of this Magnification).

Definitions 1.7. The bad news about using S instead of σ in our test statistic (*) above is that, even with a normal population X,

$$T_{n-1} \equiv \frac{(\overline{X}-\mu)}{\frac{S}{\sqrt{n}}} \quad (**)$$

is no longer normal.

Thus we need a new name for a new probability distribution.

For $n = 2, 3, 4, \ldots$, the T **random variable, with** $(n-1)$ **degrees of freedom**, denoted T_{n-1}, or just T, if there is no ambiguity, is the random variable with the distribution of (**), with X normal.

The corresponding distribution is called the t **distribution, with** $(n-1)$ **degrees of freedom** and is denoted t_{n-1}. See [3, Definition 11], for the definition of distribution of a random variable.

Historical Remarks 1.8. The t distribution was invented by W.S. Gosset, writing under the pseudonym "Student." The "t" stands for "test value."

The distribution in (*), or, more generally, the central limit theorem, due, in the generality usually used today, to Laplace, appeared in the late 1700s, while the t distribution appeared only in the early 1900s. Good ideas take a long time to evolve.

See [7] for both Gosset and Laplace.

Examples 1.9. See the "t Curve Tail Areas" table at the end of this Magnification for probabilities of the form

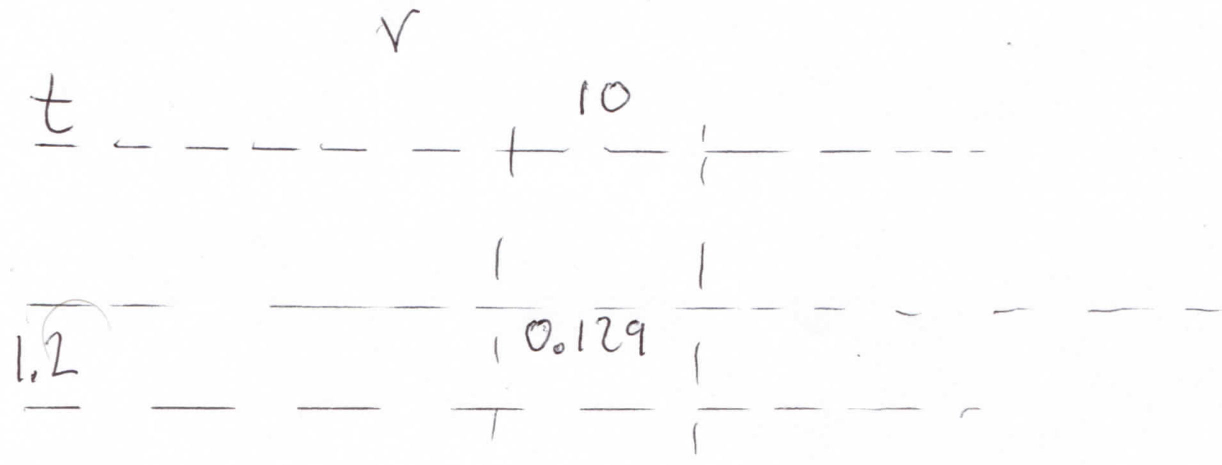
$$P(T_\nu > t) = P(T_\nu \geq t) = P(T_\nu < -t) = P(T_\nu \leq -t),$$

for t a nonnegative number, $\nu = 1, 2, 3, \ldots$. The Greek letter ν, pronounced "new," is the number of degrees of freedom.

For example, if T has 10 degrees of freedom, and we want $P(T > 1.2) = P(T_{10} > 1.2)$, then look at the intersection of the column under $\nu = 10$ and the row to the right of $t = 1.2$:

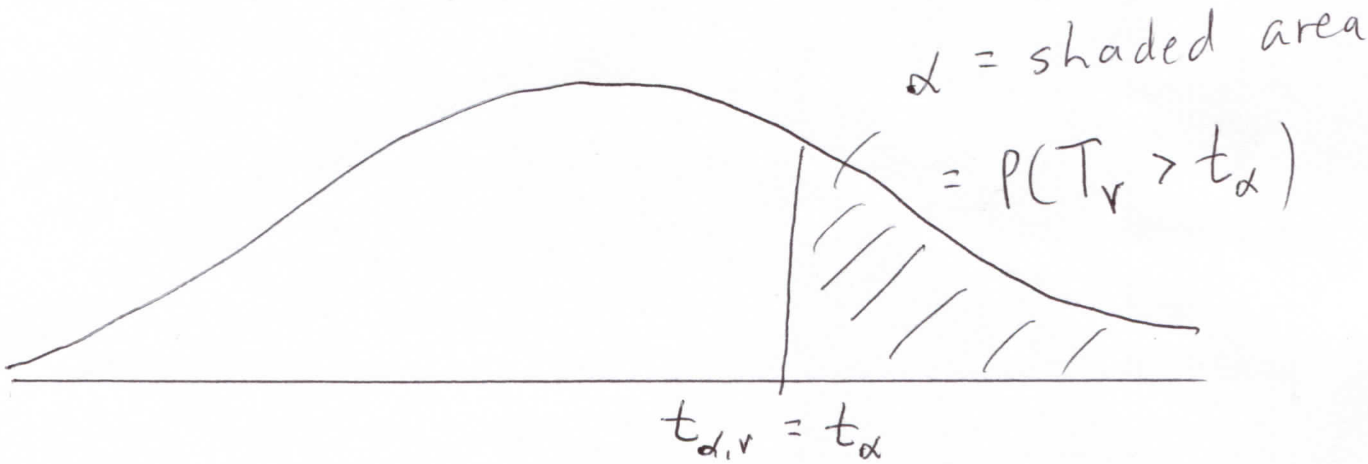
$$P(T_{10} > 1.2) = 0.129 = P(T_{10} \geq 1.2) = P(T_{10} < -1.2) = P(T_{10} \leq -1.2).$$

Here's a reproduction of the relevant part of the "t Curve Tail Areas" table.



Definition 1.10. Critical values for a T random variable are defined exactly as with Z (see [4, Chapter 2]): For α a positive number less than one, $\nu = 1, 2, 3, \ldots$, the **critical value** $t_{\alpha,\nu}$, or just t_α if there is no ambiguity, is the number such that

$$P(T_\nu > t_{\alpha,\nu}) = \alpha.$$



Examples 1.11. For some popular values of α, critical values of T are in another table at the end of this magnification, labeled "Critical Values for t Distributions" table.

For example, if we wanted the critical value $t_{0.01,26}$, we would look at the intersection of the column below $\alpha = 0.01$ and the column to the right of $\nu = 26$, giving us $t_{0.01,\nu} = 2.479$.

Here is a reproduction of the relevant part of the "Critical Values for t Distributions" table.

ν \ α	0.01
1	
2	
⋮	
26	2.479

Strategy 1.12. The random variable T_ν can be made arbitrarily close to Z by letting ν, the degrees of freedom, get arbitrarily large. Let's state this informally by saying that Z is T with ∞ degrees of freedom: $Z = T_\infty$.

In particular, we can get many Z probabilities and Z critical values by using T tables with ∞ degrees of freedom.

Example 1.13. The critical value $z_{0.001}$ may be obtained from the "t Curve Tail Areas" table with $\nu = \infty$:

$$z_{0.001} = 3.090.$$

ν \ α	0.001
⋮	
∞	3.090

2. TABLES for MEAN and PROPORTION SCENARIOS

This chapter puts all relevant information about the most popular confidence interval and hypothesis test constructions in Tables 2.3 through 2.6. In each table, results are indexed by Scenarios (1)–(12), described in the three left columns of Table 2.3, labeled "SCENARIO," "PARAMETER," and "RANDOM VARIABLE." The right-most column of Table 2.3 contains the test statistic for each of Scenarios (1)–(12).

Just as with the special case already discussed in Chapter 1 of this magnification, and in [4] and [5], summarized in Scenario (1) of Table 2.3, confidence intervals and upper and lower confidence bounds (in Table 2.4) and hypothesis tests (in Tables 2.5 and 2.6) follow from the test statistic matching the given scenario.

For example, suppose we wanted to construct a confidence interval for the difference between two population means μ_1 and μ_2 of independent normal random variables X_1 and X_2, with standard deviations unknown but equal. First we would go to Table 2.3, and scan through the second and third columns corresponding to different "SCENARIOS," eventually identifying Scenario (6) as describing our situation. Then, for our formula needed, we would go to Table 2.4, in the column under "CONFIDENCE INTERVAL," in the row that begins with (6); for anything undefined in our desired formula we would go to (6) in the right-most column of Table 2.3.

Here is a quick summary of how a test statistic leads to statistical inference; we encourage the reader to compare entries in Tables 2.4, 2.5, and 2.6 to the corresponding (meaning the same Scenario) entry in Table 2.3.

A confidence interval has the form

(estimator in numerator of test statistic) $\pm$ (critical value) $\times$ (denominator of test statistic).

An upper confidence bound has the form

(estimator in numerator of test statistic) $+$ (critical value) $\times$ (denominator of test statistic).

A lower confidence bound has the form

(estimator in numerator of test statistic) $-$ (critical value) $\times$ (denominator of test statistic).

See [4, Chapter 3] for confidence intervals and confidence bounds for a special case.

For hypothesis testing, the test statistic, with parameter θ replaced by θ_0, the null hypothesis H_0 value of θ, is either used to get a P-value, as in Table 2.5 (see Theorem 2.2), or compared to a critical value, as in Table 2.6, to decide whether to reject or not reject the null hypothesis H_0.

See [5, Tables 4.2 and 4.5] for hypothesis testing for a special case.

2.1. Some terminology needed to read Tables 2.3–2.6.

Throughout Table 2.3 (and hence Tables 2.4, 2.5, and 2.6), n is the sample size of the random sample from the random variable $X, \overline{x}$ is the sample mean, s is the sample standard deviation, μ is the population mean, and σ is the population standard deviation. The analogous relationships hold for $n_1, X_1, \overline{x}_1, s_1, \mu_1$, and σ_1 and $n_2, X_2, \overline{x}_2, s_2, \mu_2$, and σ_2.

See [4, Chapter 2], or [1], or [3, Examples 15], or [6] for normal random variables. The other random variable appearing in the third column from the left in Table 2.3 (the column is labeled "RANDOM VARIABLE") is the *binomial* (see [3, Examples 15]), which we will now define (see [1] or [6]). For n a natural number and p strictly between zero and one, the **binomial**(n, p) random variable counts the number of successes in n independent repetitions of an experiment, with p defined to be the probability of success at each repetition.

The prototype for binomial(n, p) is counting the number of heads in n flips of a coin, said coin weighted so that the probability of heads at each flip is p.

Of more interest to us is the following binomial(n, p): let p be the proportion of a population with a certain specified attribute, n the sample size of a random sample from said population (that is, we pull out n things in the population), and our binomial random variable counts the number of things in the sample with said attribute.

This is seen to be binomial when we define our repeated "experiment" to be taking something from the population, with "success" meaning the thing taken has the specified attribute.

In this setting, the **sample proportion** $\hat{p}$ is the proportion of the sample that has the specified attribute. If x is the number of things with said attribute in a sample of size n, then $\hat{p} = \frac{x}{n}$. The parameter p is called the **population proportion**.

The binomial just described, involving sampling, presumes that we either sample with replacement, or sample from a population sufficiently large that sampling without replacement is indistinguishable from sampling with replacement. The image here is measuring salt content of teacups of water taken from the ocean; the ocean does not care if the contents of the teacup are thrown back into the ocean.

See also [3, Definitions 6], for definitions of mean and proportion, both for samples and populations.

The abbreviation "df" stands for "degrees of freedom," as in the definition of the t distribution in Definitions 1.7.

Note that, in Scenarios (3) and (5), the sample sizes n, n_1, and n_2 need to be sufficiently large; this is to make our test statistic be approximated by Z with sufficient accuracy.

For Scenarios (9)–(12) we similarly want our test statistic to be approximated by Z with sufficient accuracy. Traditional conditions for this accuracy involve values of $np, n(1-p), n_1p_1, n_1(1-p_1), n_2p_2$, and $n_2(1-p_2)$, but there seems to be disagreement about which values of p to use.

We will adopt the following conventions. In Scenarios (9) and (10) we need $n\hat{p}$ and $n(1-\hat{p}) \geq 10$. In Scenarios (11) and (12), we need $n_1\hat{p}_1, n_1(1-\hat{p}_1), n_2\hat{p}_2$, and $n_2(1-\hat{p}_2)$ all ≥ 10.

Scenario (8) is the only scenario with a pair of random variables, X_1 and X_2, that are not independent. Two random variables X_1, X_2 are **paired** if they are measured on the same objects at different times or under different circumstances.

Finally, before presenting Tables 2.3–2.6, we should relate Tables 2.5 and 2.6 with the following (see [5, 3.1, 3.6, and 4.7]).

Theorem 2.2. Suppose α is a positive number less than one.

In all the scenarios of Tables 2.3–2.6, we reject H_0, at significance level α, if and only if the P-value of our data is less than or equal to α.

TABLE 2.3: TEST STATISTIC SCENARIOs

SCENARIO	PARAMETER	RANDOM VARIABLE	TEST STATISTIC
(1)	μ	X normal, σ known	$\frac{(\overline{X}-\mu)}{\frac{\sigma}{\sqrt{n}}} = Z$
(2)	μ	X arbitrary, $n > 40$	$\frac{(\overline{X}-\mu)}{\frac{S}{\sqrt{n}}} \sim Z$
(3)	μ	X normal, σ unknown	$\frac{(\overline{X}-\mu)}{\frac{S}{\sqrt{n}}} = T,$ $(n-1)\,df$
(4)	$(\mu_1 - \mu_2)$	X_1, X_2 normal and independent, σ_1 and σ_2 known	$\frac{(\overline{X}_1-\overline{X}_2)-(\mu_1-\mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}} = Z$
(5)	$(\mu_1 - \mu_2)$	X_1, X_2 independent, both n_1 and $n_2 > 40$	$\frac{(\overline{X}_1-\overline{X}_2)-(\mu_1-\mu_2)}{\sqrt{\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}}} \sim Z$
(6)	$(\mu_1 - \mu_2)$	X_1, X_2 normal and independent, σ_1 and σ_2 unknown, $\sigma_1 = \sigma_2$	$\frac{(\overline{X}_1-\overline{X}_2)-(\mu_1-\mu_2)}{S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} \sim T, (n_1+n_2-2)\,df,$ $S_p^2 \equiv \frac{1}{(n_1+n_2-2)}[(n_1-1)S_1^2 + (n_2-1)S_2^2]$
(7)	$(\mu_1 - \mu_2)$	X_1, X_2 normal and independent, σ_1 and σ_2 unknown, $\sigma_1 \neq \sigma_2$	$\frac{(\overline{X}_1-\overline{X}_2)-(\mu_1-\mu_2)}{\sqrt{\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}}} \sim T, \nu\, df,$ $\nu \equiv$ largest integer $\leq \frac{\left(\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}\right)^2}{\left[\frac{1}{(n_1-1)}\left(\frac{S_1^2}{n_1}\right)^2+\frac{1}{(n_2-1)}\left(\frac{S_2^2}{n_2}\right)^2\right]}$
(8)	$(\mu_1 - \mu_2)$	X_1, X_2 paired, $D \equiv (X_1 - X_2)$ normal, $n_1 = n_2 \equiv n$	$\frac{(\overline{D}-\mu_D)}{\frac{S_D}{\sqrt{n}}} = T,$ $(n-1)\,df$
(9)	p	X binomial(n,p)	$\frac{(\hat{p}-p)}{\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}} \sim Z$
(10)	p	X binomial(n,p)	$\frac{(\hat{p}-p)}{\sqrt{\frac{p(1-p)}{n}}} \sim Z$
(11)	$(p_1 - p_2)$	X_1 binomial(n_1,p_1), X_2 binomial(n_2,p_2)	$\frac{(\hat{p}_1-\hat{p}_2)-(p_1-p_2)}{S_{(\hat{p}_1-\hat{p}_2)}} \sim Z,$ $S_{(\hat{p}_1-\hat{p}_2)}^2 \equiv \frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}$
(12)	$(p_1 - p_2)$	X_1 binomial(n_1,p_1), X_2 binomial(n_2,p_2)	$\frac{(\hat{p}_1-\hat{p}_2)-(p_1-p_2)}{\sqrt{\hat{p}_3(1-\hat{p}_3)(\frac{1}{n_1}+\frac{1}{n_2})}} \sim Z,$ $\hat{p}_3 \equiv (\frac{(X_1+X_2)}{n_1+n_2})$

TABLE 2.4: CONFIDENCE INTERVALS and BOUNDs

For each of (1)–(12), compare to the corresponding scenario in Table 2.3.

SCENARIO	$100(1-\alpha)\%$ CONFIDENCE INTERVAL for parameter	$100(1-\alpha)\%$ UPPER CONFIDENCE BOUND for parameter	$100(1-\alpha)\%$ LOWER CONFIDENCE BOUND for parameter
(1)	$\overline{x} \pm z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$	$\overline{x} + z_{\alpha} \frac{\sigma}{\sqrt{n}}$	$\overline{x} - z_{\alpha} \frac{\sigma}{\sqrt{n}}$
(2)	$\overline{x} \pm z_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$	$\overline{x} + z_{\alpha} \frac{s}{\sqrt{n}}$	$\overline{x} - z_{\alpha} \frac{s}{\sqrt{n}}$
(3)	$\overline{x} \pm t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$	$\overline{x} + t_{\alpha} \frac{s}{\sqrt{n}}$	$\overline{x} - t_{\alpha} \frac{s}{\sqrt{n}}$
(4)	$\left[(\overline{x}_1 - \overline{x}_2) \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}\right]$	$\left[(\overline{x}_1 - \overline{x}_2) + z_{\alpha} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}\right]$	$\left[(\overline{x}_1 - \overline{x}_2) - z_{\alpha} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}\right]$
(5)	$\left[(\overline{x}_1 - \overline{x}_2) \pm z_{\frac{\alpha}{2}} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}\right]$	$\left[(\overline{x}_1 - \overline{x}_2) + z_{\alpha} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}\right]$	$\left[(\overline{x}_1 - \overline{x}_2) - z_{\alpha} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}\right]$
(6)	$\left[(\overline{x}_1 - \overline{x}_2) \pm t_{\frac{\alpha}{2}} s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}\right]$	$\left[(\overline{x}_1 - \overline{x}_2) + t_{\alpha} s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}\right]$	$\left[(\overline{x}_1 - \overline{x}_2) - t_{\alpha} s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}\right]$
(7)	$\left[(\overline{x}_1 - \overline{x}_2) \pm t_{\frac{\alpha}{2}} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}\right]$	$\left[(\overline{x}_1 - \overline{x}_2) + t_{\alpha} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}\right]$	$\left[(\overline{x}_1 - \overline{x}_2) - t_{\alpha} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}\right]$
(8)	$\overline{d} \pm t_{\frac{\alpha}{2}} \frac{s_D}{\sqrt{n}}$	$\overline{d} + t_{\alpha} \frac{s_D}{\sqrt{n}}$	$\overline{d} - t_{\alpha} \frac{s_D}{\sqrt{n}}$
(9)	$\hat{p} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$	$\hat{p} + z_{\alpha} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$	$\hat{p} - z_{\alpha} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$
(10)	More accurate than (9) above, but ugly to solve for p in $-z_{\frac{\alpha}{2}} < \frac{(\hat{p}-p)}{\sqrt{\frac{p(1-p)}{n}}} < z_{\frac{\alpha}{2}}$	More accurate than (9) above, but ugly to solve for p in $\frac{(\hat{p}-p)}{\sqrt{\frac{p(1-p)}{n}}} > -z_{\alpha}$	More accurate than (9) above, but ugly to solve for p in $\frac{(\hat{p}-p)}{\sqrt{\frac{p(1-p)}{n}}} < z_{\alpha}$
(11)	$\left[(\hat{p}_1 - \hat{p}_2) \pm z_{\frac{\alpha}{2}} S_{(\hat{p}_1 - \hat{p}_2)}\right]$	$\left[(\hat{p}_1 - \hat{p}_2) + z_{\alpha} S_{(\hat{p}_1 - \hat{p}_2)}\right]$	$\left[(\hat{p}_1 - \hat{p}_2) - z_{\alpha} S_{(\hat{p}_1 - \hat{p}_2)}\right]$
(12)	not usually used	not usually used	not usually used

TABLE 2.5: HYPOTHESIS TEST P-values

In each of (1)–(12), lower-case "z" or "t" refers to the test statistic in the corresponding scenario in Table 2.3.

SCENARIO	HYPOTHESIS TEST, one-sided	HYPOTHESIS TEST, one-sided	HYPOTHESIS TEST, two-sided
(1)	$H_0 : \mu = \mu_0$ $H_a : \mu < \mu_0$ $P\text{-value} = P(Z < z)$	$H_0 : \mu = \mu_0$ $H_a : \mu > \mu_0$ $P\text{-value} = P(Z > z)$	$H_0 : \mu = \mu_0$ $H_a : \mu \neq \mu_0$ $P\text{-value} = 2P(Z > \|z\|)$
(2)	$H_0 : \mu = \mu_0$ $H_a : \mu < \mu_0$ $P\text{-value} = P(Z < z)$	$H_0 : \mu = \mu_0$ $H_a : \mu > \mu_0$ $P\text{-value} = P(Z > z)$	$H_0 : \mu = \mu_0$ $H_a : \mu \neq \mu_0$ $P\text{-value} = 2P(Z > \|z\|)$
(3)	$H_0 : \mu = \mu_0$ $H_a : \mu < \mu_0$ $P\text{-value} = P(T < t)$	$H_0 : \mu = \mu_0$ $H_a : \mu > \mu_0$ $P\text{-value} = P(T > t)$	$H_0 : \mu = \mu_0$ $H_a : \mu \neq \mu_0$ $P\text{-value} = 2P(T > \|t\|)$
(4)	$H_0 : (\mu_1 - \mu_2) = \mu_d$ $H_a : (\mu_1 - \mu_2) < \mu_d$ $P\text{-value} = P(Z < z)$	$H_0 : (\mu_1 - \mu_2) = \mu_d$ $H_a : (\mu_1 - \mu_2) > \mu_d$ $P\text{-value} = P(Z > z)$	$H_0 : (\mu_1 - \mu_2) = \mu_d$ $H_a : (\mu_1 - \mu_2) \neq \mu_d$ $P\text{-value} = 2P(Z > \|z\|)$
(5)	$H_0 : (\mu_1 - \mu_2) = \mu_d$ $H_a : (\mu_1 - \mu_2) < \mu_d$ $P\text{-value} = P(Z < z)$	$H_0 : (\mu_1 - \mu_2) = \mu_d$ $H_a : (\mu_1 - \mu_2) > \mu_d$ $P\text{-value} = P(Z > z)$	$H_0 : (\mu_1 - \mu_2) = \mu_d$ $H_a : (\mu_1 - \mu_2) \neq \mu_d$ $P\text{-value} = 2P(Z > \|z\|)$
(6)	$H_0 : (\mu_1 - \mu_2) = \mu_d$ $H_a : (\mu_1 - \mu_2) < \mu_d$ $P\text{-value} = P(T < t)$	$H_0 : (\mu_1 - \mu_2) = \mu_d$ $H_a : (\mu_1 - \mu_2) > \mu_d$ $P\text{-value} = P(T > t)$	$H_0 : (\mu_1 - \mu_2) = \mu_d$ $H_a : (\mu_1 - \mu_2) \neq \mu_d$ $P\text{-value} = 2P(T > \|t\|)$
(7)	$H_0 : (\mu_1 - \mu_2) = \mu_d$ $H_a : (\mu_1 - \mu_2) < \mu_d$ $P\text{-value} = P(T < t)$	$H_0 : (\mu_1 - \mu_2) = \mu_d$ $H_a : (\mu_1 - \mu_2) > \mu_d$ $P\text{-value} = P(T > t)$	$H_0 : (\mu_1 - \mu_2) = \mu_d$ $H_a : (\mu_1 - \mu_2) \neq \mu_d$ $P\text{-value} = 2P(T > \|t\|)$
(8)	$H_0 : (\mu_1 - \mu_2) = \mu_d$ $H_a : (\mu_1 - \mu_2) < \mu_d$ $P\text{-value} = P(T < t)$	$H_0 : (\mu_1 - \mu_2) = \mu_d$ $H_a : (\mu_1 - \mu_2) > \mu_d$ $P\text{-value} = P(T > t)$	$H_0 : (\mu_1 - \mu_2) = \mu_d$ $H_a : (\mu_1 - \mu_2) \neq \mu_d$ $P\text{-value} = 2P(T > \|t\|)$
(9)	not usually used	not usually used	not usually used
(10)	$H_0 : p = p_0$ $H_a : p < p_0$ $P\text{-value} = P(Z < z)$	$H_0 : p = p_0$ $H_a : p > p_0$ $P\text{-value} = P(Z > z)$	$H_0 : p = p_0$ $H_a : p \neq p_0$ $P\text{-value} = 2P(Z > \|z\|)$
(11)	not usually used	not usually used	not usually used
(12)	$H_0 : (p_1 - p_2) = p_d$ $H_a : (p_1 - p_2) < p_d$ $P\text{-value} = P(Z < z)$	$H_0 : (p_1 - p_2) = p_d$ $H_a : (p_1 - p_2) > p_d$ $P\text{-value} = P(Z > z)$	$H_0 : (p_1 - p_2) = p_d$ $H_a : (p_1 - p_2) \neq p_d$ $P\text{-value} = 2P(Z > \|z\|)$

TABLE 2.6: HYPOTHESIS TEST Rejection Region, significance level α

In each of (1)–(12), lower-case "z" or "t" refers to the test statistic in the corresponding scenario in Table 2.3 and α is a positive number less than one.

SCENARIO	HYPOTHESIS TEST, one-sided	HYPOTHESIS TEST, one-sided	HYPOTHESIS TEST, two-sided
(1)	$H_0 : \mu = \mu_0$ $H_a : \mu < \mu_0$ Reject H_0 if $z \leq -z_\alpha$	$H_0 : \mu = \mu_0$ $H_a : \mu > \mu_0$ Reject H_0 if $z \geq z_\alpha$	$H_0 : \mu = \mu_0$ $H_a : \mu \neq \mu_0$ Reject H_0 if $\|z\| \geq z_{\frac{\alpha}{2}}$
(2)	$H_0 : \mu = \mu_0$ $H_a : \mu < \mu_0$ Reject H_0 if $z \leq -z_\alpha$	$H_0 : \mu = \mu_0$ $H_a : \mu > \mu_0$ Reject H_0 if $z \geq z_\alpha$	$H_0 : \mu = \mu_0$ $H_a : \mu \neq \mu_0$ Reject H_0 if $\|z\| \geq z_{\frac{\alpha}{2}}$
(3)	$H_0 : \mu = \mu_0$ $H_a : \mu < \mu_0$ Reject H_0 if $t \leq -t_\alpha$	$H_0 : \mu = \mu_0$ $H_a : \mu > \mu_0$ Reject H_0 if $t \geq t_\alpha$	$H_0 : \mu = \mu_0$ $H_a : \mu \neq \mu_0$ Reject H_0 if $\|t\| \geq t_{\frac{\alpha}{2}}$
(4)	$H_0 : (\mu_1 - \mu_2) = \mu_d$ $H_a : (\mu_1 - \mu_2) < \mu_d$ Reject H_0 if $z \leq -z_\alpha$	$H_0 : (\mu_1 - \mu_2) = \mu_d$ $H_a : (\mu_1 - \mu_2) > \mu_d$ Reject H_0 if $z \geq z_\alpha$	$H_0 : (\mu_1 - \mu_2) = \mu_d$ $H_a : (\mu_1 - \mu_2) \neq \mu_d$ Reject H_0 if $\|z\| \geq z_{\frac{\alpha}{2}}$
(5)	$H_0 : (\mu_1 - \mu_2) = \mu_d$ $H_a : (\mu_1 - \mu_2) < \mu_d$ Reject H_0 if $z \leq -z_\alpha$	$H_0 : (\mu_1 - \mu_2) = \mu_d$ $H_a : (\mu_1 - \mu_2) > \mu_d$ Reject H_0 if $z \geq z_\alpha$	$H_0 : (\mu_1 - \mu_2) = \mu_d$ $H_a : (\mu_1 - \mu_2) \neq \mu_d$ Reject H_0 if $\|z\| \geq z_{\frac{\alpha}{2}}$
(6)	$H_0 : (\mu_1 - \mu_2) = \mu_d$ $H_a : (\mu_1 - \mu_2) < \mu_d$ Reject H_0 if $t \leq -t_\alpha$	$H_0 : (\mu_1 - \mu_2) = \mu_d$ $H_a : (\mu_1 - \mu_2) > \mu_d$ Reject H_0 if $t \geq t_\alpha$	$H_0 : (\mu_1 - \mu_2) = \mu_d$ $H_a : (\mu_1 - \mu_2) \neq \mu_d$ Reject H_0 if $\|t\| \geq t_{\frac{\alpha}{2}}$
(7)	$H_0 : (\mu_1 - \mu_2) = \mu_d$ $H_a : (\mu_1 - \mu_2) < \mu_d$ Reject H_0 if $t \leq -t_\alpha$	$H_0 : (\mu_1 - \mu_2) = \mu_d$ $H_a : (\mu_1 - \mu_2) > \mu_d$ Reject H_0 if $t \geq t_\alpha$	$H_0 : (\mu_1 - \mu_2) = \mu_d$ $H_a : (\mu_1 - \mu_2) \neq \mu_d$ Reject H_0 if $\|t\| \geq t_{\frac{\alpha}{2}}$
(8)	$H_0 : (\mu_1 - \mu_2) = \mu_d$ $H_a : (\mu_1 - \mu_2) < \mu_d$ Reject H_0 if $t \leq -t_\alpha$	$H_0 : (\mu_1 - \mu_2) = \mu_d$ $H_a : (\mu_1 - \mu_2) > \mu_d$ Reject H_0 if $t \geq t_\alpha$	$H_0 : (\mu_1 - \mu_2) = \mu_d$ $H_a : (\mu_1 - \mu_2) \neq \mu_d$ Reject H_0 if $\|t\| \geq t_{\frac{\alpha}{2}}$
(9)	not usually used	not usually used	not usually used
(10)	$H_0 : p = p_0$ $H_a : p < p_0$ Reject H_0 if $z \leq -z_\alpha$	$H_0 : p = p_0$ $H_a : p > p_0$ Reject H_0 if $z \geq z_\alpha$	$H_0 : p = p_0$ $H_a : p \neq p_0$ Reject H_0 if $\|z\| \geq z_{\frac{\alpha}{2}}$
(11)	not usually used	not usually used	not usually used
(12)	$H_0 : (p_1 - p_2) = p_d$ $H_a : (p_1 - p_2) < p_d$ Reject H_0 if $z \leq -z_\alpha$	$H_0 : (p_1 - p_2) = p_d$ $H_a : (p_1 - p_2) > p_d$ Reject H_0 if $z \geq z_\alpha$	$H_0 : (p_1 - p_2) = p_d$ $H_a : (p_1 - p_2) \neq p_d$ Reject H_0 if $\|z\| \geq z_{\frac{\alpha}{2}}$

3. EXAMPLES.

See [5, HW no. 13]. Except in 3(g), assume, in each part in this chapter, that different samples are independent of each other.

3(a.) "Barking Fools" brand of dog food advertises that it will make dogs faster, on average. Test this claim, with significance level 0.05, if the average dog runs 30 miles per hour, while a sample of 100 dogs eating "Barking Fools" has an average of 32 miles per hour, standard deviation of 10 miles per hour.

3(b). "Barking Fools" brand of dog food advertises that it will make dogs faster, on average. Test this claim, with significance level 0.05, if the average dog runs 30 miles per hour, randomly chosen dogs eating "Barking Fools" are normally distributed, and a random sample of 12 dogs eating "Barking Fools" has an average of 32 miles per hour, standard deviation of 10 miles per hour.

3(c). "Barking Fools" brand of dog food advertises that it will make dogs faster, on average. Test this claim, with significance level 0.05, if the average running speed of 12 randomly chosen dogs that do not eat "Barking Fools" is 30 miles per hour, the average running speed of 15 randomly chosen dogs eating "Barking Fools" is 32 miles per hour, randomly chosen dogs that eat "Barking Fools" are normally distributed, with a standard deviation of 10 miles per hour, while randomly chosen dogs that do not eat "Barking Fools" are normally distributed, with a standard deviation of 8 miles per hour.

3(d). "Barking Fools" brand of dog food advertises that it will make dogs faster, on average. Test this claim, with significance level 0.1, if a random sample of 50 dogs that do not eat "Barking Fools" has average speed of 30 miles per hour with standard deviation 8 miles per hour, and a random sample of 100 dogs that eat "Barking Fools" has an average speed of 32 miles per hour, with standard deviation 10 miles per hour.

3(e). "Barking Fools" brand of dog food advertises that it will make dogs faster, on average. Test this claim, with significance level 0.05, if a random sample of 5 dogs that do not eat "Barking Fools" has average speed of 30 miles per hour with standard deviation 8 miles per hour, and a random sample of 4 dogs that eat "Barking Fools" has an average speed of 32 miles per hour, with standard deviation 10 miles per hour.

Assume normality of both dogs that eat "Barking Fools" and dogs that do not eat "Barking Fools." Assume also that the standard deviation of randomly chosen dogs that eat "Barking Fools" equals the standard deviation of randomly chosen dogs that do not eat "Barking Fools."

3(f). "Barking Fools" brand of dog food advertises that it will make dogs faster, on average. Test this claim, with significance level 0.05, if a sample of 5 dogs that do not eat "Barking Fools" has average speed of 30 miles per hour with standard deviation 8 miles per hour, and a sample of 4 dogs that eat "Barking Fools" has an average speed of 32 miles per hour, with standard deviation 10 miles per hour.

Assume normality of both dogs that eat "Barking Fools" and dogs that do not eat "Barking Fools."

Do not assume that the standard deviation of randomly chosen dogs that eat "Barking Fools" equals the standard deviation of randomly chosen dogs that do not eat "Barking Fools."

3(g). Nine dogs are chosen at random. They are fed "Barking Fools" dog food for the month of January, then fed only food other than "Barking Fools" dog food for the next month. The following table shows their running speeds.

Dog	1	2	3	4	5	6	7	8	9
average running speed in January	38	30	30	35	20	35	30	40	30
average running speed in February	40	25	30	25	25	40	30	30	25

Does the data suggest that "Barking Fools" makes dogs faster, on average? Assume normality where needed.

3(h). Suppose 20 percent of all dogs run faster than 30 miles per hour, while, in a random sample of 64 dogs that eat "Barking Fools" dog food, 32 dogs run faster than 30 miles per hour. Does this provide sufficient evidence to conclude, at significance level 0.1, that "Barking Fools" makes dogs faster?

3(i). Suppose, in a random sample of 100 dogs that don't eat "Barking Fools," 20 dogs run faster than 30 miles per hour, while, in a random sample of 64 dogs that eat "Barking Fools" dog food, 16 dogs run faster than 30 miles per hour. Does this provide sufficient evidence to conclude, at significance level 0.1, that "Barking Fools" makes dogs faster?

3(j). In 3(a), find a 95% lower confidence bound for the average running speed of all dogs eating "Barking Fools" dog food.

3(k). In 3(a), find a 90% confidence interval for the average running speed of all dogs eating "Barking Fools" dog food.

3(l). In 3(b), find a 95% upper confidence bound for the average running speed of all dogs eating "Barking Fools" dog food.

3(m). In 3(c), find a 90% confidence interval for the difference between the average running speed of a dog that eats "Barking Fools" and the average running speed of a dog that does not eat "Barking Fools."

3(n). In 3(e), find a 95% lower confidence bound and a 95% upper confidence bound for the difference between the average running speed of a dog that eats "Barking Fools" and the average running speed of a dog that does not eat "Barking Fools."

3(o). In 3(f), find a 95% lower confidence bound and a 95% upper confidence bound for the difference between the average running speed of a dog that eats "Barking Fools" and the average running speed of a dog that does not eat "Barking Fools."

3(p). In 3(h), find a 95% lower confidence bound for the proportion of dogs eating "Barking Fools" that run faster than 30 miles per hour.

3(q). In 3(i), find a 90% confidence interval for the difference between the proportion of dogs eating "Barking Fools" that run faster than 30 miles per hour and the proportion of dogs not eating "Barking Fools" that run faster than 30 miles per hour.

SOLUTIONS to Examples

In all examples, answers may differ by small amounts, because of different rounding or use of t tables instead of Z tables (see Strategy 1.12).

3(a) Solution. Since we are given the average running speed of all dogs, a relevant parameter, for judging the effect of "Barking Fools," is μ defined to be the average running speed of dogs eating "Barking Fools." Since the sample described measures running speed of 100 dogs eating "Barking Fools," this choice of parameter should work.

The advertisement "make dogs faster" we may interpret as $\mu > 30$, the average running speed of all dogs. As is traditional (see [5]), we make this advertising claim H_a, the alternative hypothesis. Thus our hypothesis test is

$$H_0 : \mu = 30, \quad H_a : \mu > 30.$$

Here is the information given:

$$n = 100, \overline{x} = 32, s = 10.$$

Since $n > 40$, this is Scenario (2) in Table 2.3, so our test statistic is

$$z = \frac{(32 - 30)}{\frac{10}{\sqrt{100}}} = 2.$$

We could use Table 2.5 and Theorem 2.2:

$$P\text{-value} = P(Z > 2) = 0.0228 \le 0.05 = \alpha,$$

thus we reject H_0; the data suggests, at significance level 0.05, that "Barking Fools" makes dogs run faster.

Notice that we would *not* get this conclusion, at significance level 0.01, since our P-value is greater than 0.01.

We could also have done this problem with Table 2.6:

$$z = 2 \ge 1.645 = z_{0.05},$$

thus we reject H_0, at significance level 0.05. Notice that we would again fail to reject H_0, at significance level 0.01, since $z = 2 < 2.326 = z_{0.01}$.

3(b) Solution. This is the same as (a), except that the sample size $n = 12 \le 40$, thus we are now in Scenario (3), so our test statistic is t, with $(12 - 1) = 11$ degrees of freedom:

$$t = \frac{(32 - 30)}{\frac{10}{\sqrt{12}}} \sim 0.69.$$

We could get a P-value, using Table 2.5 and Theorem 2.2:

$$P(T_{11} > 0.69) \sim P(T_{11} > 0.7) = 0.249 > 0.05 = \alpha,$$

thus we don't reject H_0; the data doesn't suggest, at significance level 0.05, that "Barking Fools" makes dogs faster.

Or we could use Table 2.6:

$$t = 0.69 < 1.796 = t_{0.05} \text{ (critical value for 11 degrees of freedom)},$$

so we don't reject H_0.

3(c) Solution. Now we are sampling from two populations, those that eat "Barking Fools," call them X_1, and those that don't eat "Barking Fools," call them X_2. More generally, let's index information with a subscript of 1 for eating "Barking Fools" and a subscript of 2 for not eating "Barking Fools."

Since our samples measure average running speeds, we choose parameters μ_1 and μ_2 for average running speeds of X_1 and X_2. The claim that "Barking Fools" makes dogs faster becomes $(\mu_1 - \mu_2) > 0$, and this becomes H_a, so that our hypothesis test is

$$H_0 : (\mu_1 - \mu_2) = 0, \quad H_a : (\mu_1 - \mu_2) > 0.$$

So far we have five possible scenarios from Table 2.3, Scenarios (4)–(8). To choose, we must translate the information given:

$$\overline{x}_1 = 32, n_1 = 15, \sigma_1 = 10, \overline{x}_2 = 30, n_2 = 12, \sigma_2 = 8.$$

This looks like Scenario(4), thus our test statistic is

$$z = \frac{(32 - 30) - 0}{\sqrt{\frac{10^2}{15} + \frac{8^2}{12}}} \sim 0.58.$$

This gives us a P-value of

$$P(Z > 0.58) = 0.2810 > 0.05 = \alpha,$$

thus we don't reject H_0; the data does not support the advertisement.

3(d) Solution. Same hypothesis test as in (c), but different information and a different significance level:

$$\overline{x}_1 = 32, n_1 = 100, s_1 = 10, \overline{x}_2 = 30, n_2 = 50, s_2 = 8.$$

Because of our large n_1 and n_2, we have Scenario(5), thus our test statistic is

$$z = \frac{(32 - 30) - 0}{\sqrt{\frac{10^2}{100} + \frac{8^2}{50}}} \sim 1.32.$$

Now our P-value is

$$P(Z > 1.32) = 0.0934 \leq 0.1 = \alpha,$$

so we reject H_0: the data suggests, at significance level 0.1, that the advertisement is correct.

The P-value tells us how close we came to not supporting the advertising claim; any P-value greater than 0.1 would mean failure for our advertisement, thus our P-value of 0.0934 barely suffices.

3(e) Solution. Same hypothesis test as in (c) and (d), but let's worry about our current information:

$$\overline{x}_1 = 32, n_1 = 4, s_1 = 10, \overline{x}_2 = 30, n_2 = 5, s_2 = 8, \sigma_1 = \sigma_2.$$

That equality of σs tells us we are in Scenario (6).

Our test statistic now has a t distribution, with $(4 + 5 - 2) = 7$ degrees of freedom. Embedded in our test statistic for Scenario (6) is

$$s_p^2 = \frac{1}{4 + 5 - 2}\left[(4 - 1)10^2 + (5 - 1)8^2\right] \sim 79,$$

so that our test statistic is

$$t = \frac{(32 - 30)}{\sqrt{79}\sqrt{\frac{1}{4} + \frac{1}{5}}} \sim 0.34.$$

We could use the P-value

$$P(T_7 > 0.34) \sim 0.386 > 0.05 = \alpha,$$

thus we don't reject H_0; data doesn't suggest "Barking Fools" makes dogs faster.

Alternatively, the critical value $t_{0.05}$, for 7 degrees of freedom, is 1.895, thus

$$t \sim 0.34 < 1.895 = t_{0.05},$$

implying we don't reject H_0.

3(f) Solution. This is the same as (e), except that $\sigma_1 \neq \sigma_2$, so we are thrust into Scenario (7). This begins with an ugly calculation of ν, the degrees of freedom:

$$\frac{\left(\frac{10^2}{4}+\frac{8^2}{5}\right)^2}{\left[\frac{1}{4-1}(\frac{10^2}{4})^2+\frac{1}{5-1}(\frac{8^2}{5})^2\right]} \sim 5.7,$$

thus we choose $\nu \equiv 5$ degrees of freedom, for our t test statistic:

$$t = \frac{(32-30)-0}{\sqrt{\frac{10^2}{4}+\frac{8^2}{5}}} \sim 0.3,$$

so our P-value is

$$P(T_5 > 0.3) = 0.388 > 0.05 = \alpha,$$

so we don't reject H_0; the data is not sufficient, at significance level 0.05, to support the advertised claim.

3(g) Solution. This is the same hypothesis test as in (c), (d), (e), and (f), but we now have paired data (see 2.1), thus we are in Scenario (8), which tells us to look at $D \equiv (X_1 - X_2)$, with the following data.

dog	1	2	3	4	5	6	7	8	9
d	-2	5	0	10	-5	-5	0	10	5

We will use the Computational Formula 1.3 to get $\overline{d}$ and s_d:

$$(d_1 + d_2 + \cdots + d_9) = 18, \;\; (d_1^2 + d_2^2 + \cdots + d_9^2) = 304$$

implies that

$$\overline{d} = \frac{18}{9} = 2, \;\; s_d^2 = \frac{1}{8}\left[304 - \frac{1}{9}(18)^2\right] = 33.5.$$

Thus our test statistic, with $(9-1) = 8$ degrees of freedom, is

$$t = \frac{(2-0)}{\sqrt{\frac{33.5}{9}}} \sim 1.0,$$

giving us a P-value of

$$P(T_8 > 1.0) = 0.173.$$

When no significance level is given, it is "customary" (an important deal maker in many statistical circles) to use either 0.01 or 0.05 for a significance level.

Since our P-value is greater than both 0.01 and 0.05, we do not reject H_0; at significance level 0.01 and 0.05, the data does not imply that "Barking Fools" makes dogs run faster.

3(h) Solution. Just so we can use the information about the proportion of dogs running faster than 30 miles per hour, let's interpret "makes dogs run faster" as producing a higher proportion that run faster than 30 miles per hour.

Define p to be the proportion of dogs eating "Barking Fools" that run faster than 30 miles per hour. Then our hypothesis test is

$$H_0 : p = 0.2, \;\; H_a : p > 0.2.$$

We have

$$n = 64, x = 32, \hat{p} = \frac{32}{64} = 0.5,$$

and we are in Scenario (10), with test statistic

$$z = \frac{(0.5-0.2)}{\sqrt{\frac{(0.2)(0.8)}{64}}} = 6 \geq 1.282 = z_{0.1},$$

thus, by Table 2.6, we reject H_0; at significance level 0.1, the data provides sufficient evidence that "Barking Fools" makes dogs run faster.

3(i) Solution. As with (h), let's have "faster" mean a larger proportion that run faster than 30 miles per hour. Letting X_1 count the number of dogs eating "Barking Fools" that run faster than 30 miles per hour, X_2 likewise for dogs not eating "Barking Fools," our hypothesis test is then

$$H_0 : (p_1 - p_2) = 0, \quad H_a : (p_1 - p_2) > 0.$$

We have

$$n_1 = 64, x_1 = 16, \hat{p}_1 = \frac{16}{64} = 0.25, n_2 = 100, x_2 = 20, \hat{p}_2 = \frac{20}{100} = 0.2,$$

and we are in Scenario (12).

For our test statistic, we first need

$$\hat{p}_3 = \frac{(16+20)}{(64+100)} \sim 0.22,$$

so that

$$z = \frac{(0.25 - 0.2) - 0}{\sqrt{0.22(1-0.22)(\frac{1}{64} + \frac{1}{100})}} \sim 0.75$$

makes a P-value of

$$P(Z > 0.75) = 0.2266 > 0.1 = \alpha,$$

thus we don't reject H_0; the data does not suggest, at significance level 0.1, that "Barking Fools" makes dogs faster.

3(j) Solution. From (a) and Table 2.4, Scenario (2), since $(1-\alpha) = 0.95$, so that $\alpha = 0.05$, our lower confidence bound is

$$32 - z_{0.05}\frac{10}{\sqrt{100}} = 32 - 1.645 = 30.355.$$

3(k) Solution. Again from (a) and Table 2.4, Scenario (2), since $(1-\alpha) = 0.9$, so that $\frac{\alpha}{2} = 0.05$, our confidence interval is

$$32 \pm z_{0.05}\frac{10}{\sqrt{100}} = 32 \pm 1.645 = (30.355, 33.645).$$

3(l) Solution. From (b), we want Scenario (3), with 11 degrees of freedom; from (a), we have $\alpha = 0.05$, thus we want

$$32 + t_{0.05}\frac{10}{\sqrt{12}} = 32 + (1.796)\frac{10}{\sqrt{12}} \sim 37.18.$$

3(m) Solution. From (c) and Table 2.4, Scenario (4), this is

$$(32-30) \pm z_{0.05}\sqrt{\frac{10^2}{15} + \frac{8^2}{12}} = 2 \pm 5.7 = (-3.7, 7.7).$$

3(n) Solution. From (e), we are talking about Scenario (6), with 7 degrees of freedom. We also use calculations from (e):
Lower confidence bound:

$$(32-30) - t_{0.05}s_p\sqrt{\frac{1}{4} + \frac{1}{5}} = 2 - 1.895\sqrt{79(0.45)} \sim -9.3;$$

Upper confidence bound:

$$(32-30) + t_{0.05}s_p\sqrt{\frac{1}{4} + \frac{1}{5}} = 2 + 1.895\sqrt{79(0.45)} \sim 13.3.$$

3(o) Solution. From (f), we have $\nu = 5$ degrees of freedom, so

$$t_{0.05} = 2.015.$$

Use Table 2.4, Scenario (7):

Lower confidence bound:

$$(32-30) - 2.015\sqrt{\frac{10^2}{4}+\frac{8^2}{5}} = 2 - 12.4 = -10.4;$$

Upper confidence bound:

$$2 + 12.4 = 14.4.$$

3(p) Solution. From Scenario (9) in Table 2.4 and the data from (h), we want

$$0.5 - z_{0.05}\sqrt{\frac{0.5(1-0.5)}{64}} = 0.5 - 1.645 \times 0.0625 \sim 0.4.$$

3(q) Solution. From Scenario (11) in Table 2.4 and the data from (i), we want

$$(0.25-0.2) \pm 1.645\sqrt{\frac{0.25(1-0.25)}{64}+\frac{0.2(1-0.2)}{100}} \sim 0.05 \pm 0.11 = (-0.06, 0.16).$$

HOMEWORK

Except for paired data, assume in each problem that pairs of random variables are independent.

1. Compare with [5, HW no. 16], and the Examples in Chapter 3 of this magnification.

(a). A lotion called SkinTemp advertises that its use will change people's skin temperature, on average. Test this advertisement, at significance level 0.01, if the average skin temperature of 100 people using SkinTemp is 97 degrees, with a standard deviation of 5 degrees. Assume that the skin temperature of a randomly chosen person not using SkinTemp has a mean of 99 degrees.

(b). A lotion called SkinTemp advertises that its use will change people's skin temperature, on average. Test this advertisement, at significance level 0.01, if the average skin temperature of 9 people using SkinTemp is 97 degrees, with a standard deviation of 5 degrees. Assume that the skin temperature of a randomly chosen person not using SkinTemp has a mean of 99 degrees. Also assume that the skin temperature of people using SkinTemp is normally distributed.

(c). A lotion called SkinTemp advertises that its use will change people's skin temperature, on average. Test this advertisement, at significance level 0.01, if the average skin temperature of 9 people using SkinTemp is 97 degrees and the average skin temperature of 6 people not using SkinTemp is 99 degrees.

Assume that the skin temperature of people using SkinTemp is normally distributed with a standard deviation of 5 degrees and the skin temperature of people not using SkinTemp is normally distributed with a standard deviation of 2 degrees.

(d). A lotion called SkinTemp advertises that its use will change people's skin temperature, on average. Test this advertisement, at significance level 0.01, if the average skin temperature of 50 people using SkinTemp is 97 degrees, with a standard deviation of 5 and the average skin temperature of 100 people not using SkinTemp is 99 degrees, with a standard deviation of 2.

(e). A lotion called SkinTemp advertises that its use will change people's skin temperature, on average. Test this advertisement, at significance level 0.01, if the average skin temperature of 9 people using SkinTemp is 97 degrees, with a standard deviation of 5 and the average skin temperature of 6 people not using SkinTemp is 99 degrees, with a standard deviation of 2.

Assume normality both of people who use SkinTemp and people who do not Also assume that the standard deviation of randomly chosen people using SkinTemp equals the standard deviation of randomly chosen people not using SkinTemp.

(f). A lotion called SkinTemp advertises that its use will change people's skin temperature, on average. Test this advertisement, at significance level 0.01, if the average skin temperature of 9 people using SkinTemp is 97 degrees, with a standard deviation of 5 and the average skin temperature of 6 people not using SkinTemp is 99 degrees, with a standard deviation of 2.

Assume normality both of people who use SkinTemp and people who do not. Do not assume that the standard deviation of randomly chosen people using SkinTemp equals the standard deviation of randomly chosen people not using SkinTemp.

(g). A lotion called SkinTemp advertises that its use will change people's skin temperature, on average. Test this advertisement, at significance level 0.01, if four people use SkinTemp for a week in June and don't use SkinTemp for a week a year later, producing the following skin temperatures.

Person	1	2	3	4
skin temperature in earlier year	100	99	98	99
skin temperature in subsequent year	98	99	97	99

Assume normality where needed.

(h). A lotion called SkinTemp advertises that its use will change people's skin temperature, on average. Test this advertisement, at significance level 0.01, if 20 percent of all people who do not use SkinTemp have skin temperatures above 98, while 50 percent of 100 people who use SkinTemp have skin temperatures above 98.

(i). A lotion called SkinTemp advertises that its use will change people's skin temperature, on average. Test this advertisement, at significance level 0.01, if 50 percent of 100 people who use SkinTemp have skin temperatures above 98, while 20 percent of 80 people who do not use SkinTemp have skin temperatures above 98.

(j). In (b), find a 99% confidence interval for the average skin temperature of people using SkinTemp.

(k). In (b), find a 99% upper confidence bound for the average skin temperature of people using SkinTemp.

(l). In (d), find a 99% confidence interval for the difference between the average skin temperature of people using SkinTemp and the average skin temperature of people not using SkinTemp.

(m). In (h), find a 99% confidence interval for the proportion of people using SkinTemp that have skin temperatures above 98 degrees.

(n). In (h), find a 99% lower confidence bound for the proportion of people using SkinTemp that have skin temperatures above 98 degrees.

(o). In (i), find a 99% confidence interval for the difference between the proportion of people using SkinTemp that have skin temperatures above 98 degrees and the proportion of people not using SkinTemp that have skin temperatures above 98 degrees.

2. 100 ephemerabugs have an average lifetime of 0.81 days with standard deviation of 0.34 days. Calculate a 99% confidence interval for the true average lifetime of all ephemrabugs. Also get a 99% upper confidence bound and 99% lower confidence bound for this average.

3. SAME as no. 2, except replace 99% with 90%.

4. Use the data in no. 2 to test, at significance level 0.01, the claim that the average lifetime of all ephemera bugs is less than 0.87 days.

5. SAME as no. 4, except at significance level 0.1.

6. Out of 500 people sampled, 70 are bald. Get a 99.9% lower confidence bound for the proportion of all people that are bald.

7. Use the data in no. 6 to test, at significance 0.001, the assertion that more than 5% of all people are bald.

8. Out of 400 children, 15 percent are overexcited. Get a 95% confidence interval for the proportion of all children who are overexcited.

9. In no. 8, is there compelling evidence, at significance level 5%, to believe that the proportion of all children who are overexcited is different than 12 percent?

10. Assume sugar content in a randomly chosen "Big Gulp (BG)" is normal. Your younger brother buys five BGs and measures an average sugar content of 3 grams, standard deviation of 0.6 grams.
Get a 90% confidence interval for the average sugar content of all BGs.

11. Suppose you have 25 slimemolds in your kitchen. Their average weight is 4 ounces, their standard deviation 1.5 ounces. Assume slimemold weight is normally distributed.
Estimate the true average weight of all slimemolds in a way that conveys information about precision and reliability (this means confidence interval, either 99 or 95 percent).

12. The drying time of a randomly chosen rag is normal, with standard deviation 10 minutes. Test the claim that the average drying time of all rags is more than 75 minutes, if the average drying time for a sample of 16 rags is 80 minutes.

Get a P-value.

13. SAME as no. 12, except the standard deviation is of the sample.

14. Suppose the recommended Vitamin Z per day is 15 mg. In a sample of 100 people, the daily Vitamin Z per day averaged 13 mg., with a standard deviation of 8 mg. Does this data indicate that daily Vitamin Z intake (on average) is significantly different than recommended?

15. In a sample of 400 wolverines, 24 of them are rabid. Does this provide compelling evidence, at significance level 0.05, that fewer than 10% of all wolverines are rabid?

16. Among 60 Martians we've captured, 15 of them have more than five fingers on each hand. Can we conclude, at significance level 0.1, that more than 20 percent of all Martians have more than five fingers on each hand?

17. Suppose the heights of ten randomly chosen Earthlings adds up to 25, while the heights of seven Martians adds up to 14.

Assume that heights of randomly chosen Earthlings is normally distributed with a standard deviation of 2, and the heights of randomly chosen Martians is normally distributed with a standard deviation of 5.

(a) Find a 90% confidence interval for the difference, in average height, between Earthlings and Martians.

(b) Test, at significance level 0.1, the claim that Earthlings are taller than Martians, on average.

18. SAME as no. 17, except the standard deviations refer to the samples rather than the population, with the population standard deviations assumed to be equal.

19. Suppose, for terminology as in Table 2.3, $n_1 = 6, n_2 = 12, \overline{x}_1 = 8, \overline{x}_2 = 6, s_1 = 3$, and $s_2 = 5$. Assume normality where needed.

(a) Get a 99% confidence interval for $(\mu_1 - \mu_2)$.

(b) Test $H_0 : (\mu_1 - \mu_2) = (-1), H_a : (\mu_1 - \mu_2) > (-1)$ at significance level 0.1.

20. A wonder drug claims to make you stronger—more specifically, able to military press at least five pounds more (on average)—10 minutes after taking it.

Letting X_2 = number of pounds you can military press before taking drug, X_1 = same 10 minutes after taking drug, test the claim with the following data for nine people:

person	1	2	3	4	5	6	7	8	9
x_1	55.85	58.84	62.05	55.74	50.89	71.05	55.01	54.96	57.47
x_2	48.23	50.84	52.96	49.68	49.50	54.98	46.61	46.07	54.59

Assume normality of $D \equiv (X_1 - X_2)$.

21. Brand X detergent left grease stains on 15 out of 100 shirts washed, while Brand Y left grease stains on 24 out of 400.

(a) Find a 95% confidence interval for the difference between the proportion of all shirts left with grease stains by X and the proportion of all shirts left with grease stains by Y.

(b) At significance level 0.05, does the data imply that Brand X leaves a higher proportion of grease stains than Brand Y?

22. A survey states that 55 out of 100 children would like a candy store built, while 210 of 400 adults would like a candy store built. Is there a significant difference between the proportion of children and the proportion of adults who would like a candy store built?

23. Here are the number of calories of randomly selected "SweetStuff" (SS) candies:

$$12, 11, 9, 12, 16.$$

The manufacturer of SS claims the average number of calories among all SS is less than 14. Test this claim, at significance level 0.1, under the assumption that SS calories is normally distributed.

24. The average mass of 10 wombats in Tasmania is 80 pounds with a standard deviation of 4, and the average mass of 20 wombats in mainland Australia is 75 pounds with a standard deviation of 5.

Find a 99% confidence interval for the difference between the average weight of wombats in Tasmania and the average weight of wombats in mainland Australia.

Assume the distributions of wombats in Tasmania and wombats in mainland Australia are normal, with equal standard deviations.

25. 15 out of 25 fish eaters solve a certain puzzle, while only 10 out of 20 non-fish eaters solve this puzzle. Does this data suggest, at significance level 0.05, that fish makes you smarter?

26. Suppose a sample of 100 Neptunian scumslugs has a mean length of 6.2 inches and a standard deviation of 2 inches, while a sample of 200 Plutonian scumslugs has a mean length of 5.7 inches and a standard deviation of 3 inches.

(a) Test the assertion that the average length of all Neptunian scumslugs is different than the average length of all Plutonian scumslugs, at significance level 0.01.

(b) Find a 99% confidence interval for the difference between the average length of all Neptunian scumslugs and the average length of all Plutonian scumslugs.

Assume normality where needed.

27. Eight randomly chosen fish eaters have an average score of 65, while ten randomly chosen non-fish eaters have an average score of 60. Does this data suggest that fish makes you smarter? Assume that the standard deviation of all fish eater scores is 3 and the standard deviation of all non-fish eater scores is 5.

Assume normality where needed and use $\alpha = 0.01$.

28. I collect 400 wolverines at random, and find that 28 of them are rabid.

(a) Find a 95% upper confidence bound for the proportion of all wolverines that are rabid.

(b) Test, at significance level 0.05, the theory that fewer than 10 percent of all wolverines are rabid.

29. Suppose 30% of all people like onions on their hamburgers. Among a sample of 100 college students, I find that 42 of them like onions on their hamburgers. Does this provide strong evidence that the proportion of all college students who like onions on their hamburgers exceeds the proportion for all people?

30. Data on the cholesterol levels of 6 wolverines give an average of 90 and a standard deviation of 12. Find a 95% percent confidence interval for the average cholesterol level of all wolverines. Assume normality of cholesterol levels.

31. Here are the test scores of 5 Martians: 114, 100, 104, 129, 153. Assume Martian test scores throughout the solar system are normally distributed, with a standard deviation of 15. Is there evidence that the mean test score of all Martians differs from 100?

32. Here are the weights, in grams, of 9 wolverines:

$$31, 31, 43, 40, 20, 35, 30, 30, 10.$$

Assume that the weight of a randomly chosen wolverine is normally distributed, with a standard deviation of 7. Give a 99% confidence interval for the average weight of all wolverines

33. The lengths of five scumslugs, in inches, is

$$56, 68, 52, 24, 50.$$

Assuming normality, what is a 99% confidence interval for the average length of all scumslugs?

34. The worldwide average score on a math exam is 70. Sixteen randomly chosen fish eaters have an average score of 85, with a standard deviation of 3. Does this data suggest that fish makes you smarter? Assume normality where needed and use $\alpha = 0.01$.

HOMEWORK SOLUTIONS

In all examples, answers may differ by small amounts, because of different rounding or use of t tables instead of Z tables (see Strategy 1.12).

1. Let's write "ST" for "SkinTemp."

(a). Let μ be the average skin temperature of all people using ST. Our hypothesis test is

$$H_0 : \mu = 99, \;\; H_a : \mu \neq 99.$$

Here's our data:

$$\overline{x} = 97, s = 5, n = 100,$$

thus we are in Scenario (2), with test stat

$$z = \frac{97 - 99}{\frac{5}{\sqrt{100}}} = -4,$$

so that

$$|z| = 4 \geq 2.576 = z_{0.005} = z_{\frac{\alpha}{2}},$$

implying (Table 2.6) that we reject H_0; the data suggests, at significance level 0.01, that ST changes people's skin temperature, on average.

(b). This is the same as (a), except that $n = 9$ and we assume normality. We are now in Scenario (3), with test stat

$$t = \frac{97 - 99}{\frac{5}{\sqrt{9}}} = -1.2,$$

where t has 8 degrees of freedom, implying that $t_{0.005} = 3.355$, so that

$$|t| = 1.2 \leq 3.355 = t_{0.005} = t_{\frac{\alpha}{2}},$$

thus Table 2.6 says we do not reject H_0; the data does not suggest, at significance level 0.01, that ST changes people's skin temperature, on average.

OR, we could've looked at the P-value

$$2P(T_8 > 1.2) = 2 \times 0.132 = 0.264 > 0.01 = \alpha,$$

thus (see Table 2.5) we don't reject H_0.

(c). Now we have samples from two populations; let's denote X_1 for people using ST, X_2 for people not using ST. To test different population means, we set up the hypothesis test

$$H_0 : (\mu_1 - \mu_2) = 0, \;\; H_a : (\mu_1 - \mu_2) \neq 0.$$

Here's our data:

$$\overline{x}_1 = 97, \sigma_1 = 5, n_1 = 9, \overline{x}_2 = 99, \sigma_2 = 2, n_2 = 6.$$

Also X_1 and X_2 are normal. We are in Scenario (4), so we look at the test stat

$$z = \frac{(97 - 99) - 0}{\sqrt{\frac{5^2}{9} + \frac{2^2}{6}}} \sim -1.08$$

so that

$$|z| = 1.08 < 2.576 = z_{0.005} = z_{\frac{\alpha}{2}},$$

meaning we don't reject H_0; the data does not imply, at significance level 0.01, that ST changes people's skin temperatures, on average.

OR, with P-values,

$$P\text{-value} = 2P(Z > 1.08) = 2 \times 0.1401 > 0.01 = \alpha,$$

so we don't reject H_0.

(d). This is the same as (c), except we have $s_1 = 5$ instead of $\sigma_1 = 5$, and $s_2 = 2$ instead of $\sigma_2 = 2$, and $n_1 = 50, n_2 = 100$.

Those values of n_1 and n_2 put us in Scenario (5), with test stat

$$z = \frac{(97 - 99) - 0}{\sqrt{\frac{5^2}{50} + \frac{2^2}{100}}} \sim -2.72.$$

We could use rejection regions (Table 2.6):

$$|z| = 2.72 \geq 2.576 = z_{0.005} = z_{\frac{\alpha}{2}},$$

so we reject H_0; the data suggests, at significance level 0.01, that ST changes people's skin temperature, on average.

OR we could use P-values (Table 2.5):

$$P\text{-value} = 2P(Z > 2.72) = 2 \times 0.0033 = 0.0066 \leq 0.01,$$

thus we reject H_0.

(e). This is the same as (d), except for different sample sizes $n_1 = 9$ and $n_2 = 6$; also we are told that X_1 and X_2, from (c), are normal with equal standard deviations $\sigma_1 = \sigma_2$. This puts us in Scenario (6), where we need

$$s_p^2 = \frac{1}{9 + 6 - 2}\left[(9-1)5^2 + (6-1)2^2\right] \sim 16.9.$$

Our test statistic is

$$t = \frac{(97 - 99) - 0}{\sqrt{16.9}\sqrt{\frac{1}{9} + \frac{1}{6}}} \sim -0.92,$$

with $(9 + 6 - 2) = 13$ degrees of freedom, thus $t_{0.005} = 3.012$, so that $|t| < t_{0.005}$, implying (see Table 2.6) that we do not reject H_0; the data does not suggest, at significance level 0.01, the truth of the advertised claim.

(f). This is the same as (e), except we do *not* assume $\sigma_1 = \sigma_2$. This forces us into Scenario (7), with test stat

$$t = \frac{(97 - 99) - 0}{\sqrt{\frac{5^2}{9} + \frac{2^2}{6}}} \sim -1.08.$$

But the degrees of freedom in this scenario require calculating

$$\frac{(\frac{5^2}{9} + \frac{2^2}{6})^2}{\left[(\frac{1}{9-1})(\frac{5^2}{9})^2 + (\frac{1}{6-1})(\frac{2^2}{6})^2\right]} \sim 11.3,$$

so that our t distribution has 11 degrees of freedom, implying that $t_{0.005} = 3.106$, and we have the same conclusion as in (e).

(g). This is paired data, so we look at $D \equiv (X_1 - X_2)$ in Scenario (8):

Person	1	2	3	4
$x_1 \equiv$ skin temperature in earlier year	100	99	98	99
$x_2 \equiv$ skin temperature in subsequent year	98	99	97	99
$d = (x_1 - x_2)$	2	0	1	0

Let's use the Computational Formula 1.3 to get $\overline{d}$ and s_d:

$$(d_1 + d_2 + d_3 + d_4) = 3, \quad (d_1^2 + d_2^2 + d_3^2 + d_4^2) = 5,$$

so

$$\overline{d} = \frac{3}{4} = 0.75, \quad s_d^2 = \frac{1}{3}\left[5 - (\frac{1}{4})(3^2)\right] \sim 0.92.$$

Our hypothesis test is

$$H_0 : \mu_D = 0, \quad H_a : \mu_D \neq 0,$$

with test stat

$$t = \frac{0.75 - 0}{\frac{\sqrt{0.92}}{\sqrt{4}}} \sim 1.56;$$

We have $(4 - 1) = 3$ degrees of freedom, so $t_{0.005} = 5.84$, thus $|t| < t_{0.005}$, with the same negative result as in (e) and (f).

(h). Here we are comparing proportions, one of them population, one of them sample . Let p be the proportion of all ST users who have skin temperatures above 98. The claim here is that ST users are different than people who do not use ST; this means $p \neq 0.2$, the proportion of all people not using ST who have skin temperatures above 98. Thus our hypothesis test is

$$H_0 : p = 0.2, \quad H_a : p \neq 0.2.$$

We are in Scenario (10), with

$$n = 100, x = 50, \hat{p} = \frac{50}{100} = 0.5,$$

so our test stat is

$$z = \frac{0.5 - 0.2}{\sqrt{\frac{0.2(1-0.2)}{100}}} = 7.5 > 2.576 = z_{0.005},$$

thus we reject H_0; at significance level 0.01, the data suggests that ST changes people's skin temperature, on average.

(i). Now we have data (sample proportions) from both ST users and non-ST users. Using the subscript 1 for ST users and 2 for non-ST users, we have

$$n_1 = 100, x_1 = 50, \hat{p}_1 = \frac{50}{100} = 0.5, n_2 = 80, x_2 = 16, \hat{p}_2 = \frac{16}{80} = 0.2.$$

Denote by p_1 the proportion of all ST users with skin temperatures above 98, by p_2 the proportion of all non-ST users with skin temperatures above 98, then our hypothesis test is

$$H_0 : (p_1 - p_2) = 0, \quad H_a : (p_1 - p_2) \neq 0,$$

and we are in Scenario (12), thus we need

$$\hat{p}_3 = \frac{50 + 16}{100 + 80} \sim 0.367,$$

and our test stat is

$$z = \frac{(0.5 - 0.2) - 0}{\sqrt{0.367(1 - 0.367)(\frac{1}{100} + \frac{1}{80})}} \sim 4.15 > 2.576 = z_{0.005}$$

so that, as in (h), we have the same conclusion as in (h).

(j). From the data in the solution of (b), using $t_{0.005} = 3.355$ (8 degrees of freedom), our interval is

$$97 \pm 3.355\frac{5}{\sqrt{9}} = 97 \pm 5.59 = (91.41, 102.59).$$

(k). Now we need $t_{0.01} = 2.896$ (still 8 degrees of freedom), so our upper bound is

$$97 + 2.896\frac{5}{\sqrt{9}} \sim 101.8.$$

(l). We are in Scenario (5), Table 2.4, so we need $z_{0.005} = 2.576$; using data from (d), our interval is

$$(97 - 99) \pm 2.576\sqrt{\frac{5^2}{50} + \frac{2^2}{100}} \sim -2 \pm 1.89 = (-3.89, -0.11).$$

(m). From Scenario (9) and (h), with $z_{0.005} = 2.576$:

$$0.5 \pm 2.576\sqrt{\frac{0.5(1-0.5)}{100}} \sim 0.5 \pm 0.13 = (0.37, 0.63).$$

(n). Again from Scenario (9) and (h), now with $z_{0.01} = 2.326$:

$$0.5 - 2.326\sqrt{\frac{0.5(1-0.5)}{100}} \sim 0.3837.$$

(o). From Scenario (11) and (i), with $z_{0.005} = 2.576$:

$$(0.5-0.2) \pm 2.576\sqrt{\frac{0.5(1-0.5)}{100} + \frac{0.2(1-0.2)}{80}} \sim 0.3 \pm 0.173 = (0.127, 0.473).$$

2. This is Scenario (2), Table 2.4, with

$$\overline{x} = 0.81, s = 0.34, n = 100, (1-\alpha) = 0.99,$$

so that $z_\alpha = z_{0.01} = 2.326$ and $z_{\frac{\alpha}{2}} = z_{0.005} = 2.576$.

The confidence interval is

$$0.81 \pm 2.576(\frac{0.34}{\sqrt{100}}) = 0.81 \pm 0.088 = (0.722, 0.898).$$

The upper confidence bound is

$$0.81 + 2.326(\frac{0.34}{\sqrt{100}}) = 0.889.$$

The lower confidence bound is

$$0.81 - 2.326(\frac{0.34}{\sqrt{100}}) = 0.731.$$

3. Now $(1-\alpha) = 0.9$, thus $z_\alpha = z_{0.1} = 1.282$ and $z_{\frac{\alpha}{2}} = z_{0.05} = 1.645$.

The confidence interval is

$$0.81 \pm 1.645(\frac{0.34}{\sqrt{100}}) = (0.754, 0.866).$$

The upper confidence bound is

$$0.81 + 1.282(\frac{0.34}{\sqrt{100}}) = 0.854.$$

The lower confidence bound is

$$0.81 - 1.282(\frac{0.34}{\sqrt{100}}) = 0.766.$$

4. We are still in Scenario (2), but now we'd like Table 2.5.

Let μ be average lifetime of all ephemera bugs. Our hypothesis test is

$$H_0 : \mu = 0.87, \;\; H_a : \mu < 0.87,$$

with data (from no. 2)

$$\overline{x} = 0.81, s = 0.34, n = 100,$$

so that our test stat is

$$z = \frac{0.81 - 0.87}{\frac{0.34}{\sqrt{100}}} \sim -1.76,$$

producing a P-value of

$$P(Z < -1.76) = P(Z > 1.76) = 0.0392 > 0.01 = \alpha,$$

thus we do not reject H_0; data is insufficient to conclude, at significance level 0.01, that the average lifetime of these bugs is less than 0.87 days.

5. Everything except α, the significance level, is the same as no. 4; now compare the P-value of 0.0392 to $\alpha = 0.1$; since the P-value is less than or equal to α, we reject H_0; data is sufficient to conclude, at significance level 0.1, that the average lifetime of these bugs is less than 0.87 days.

6. Let p be the proportion we crave. This is Scenario (9), in Table 2.4, with data

$$x = 70, n = 500, \hat{p} = \frac{70}{500} = 0.14, (1-\alpha) = 0.999,$$

so that $z_\alpha = z_{0.001} = 3.090$, and our lower confidence bound is

$$0.14 - 3.090\left(\sqrt{\frac{0.14(1-0.14)}{500}}\right) \sim 0.092.$$

7. With p as in no. 6, our hypothesis test is

$$H_0 : p = 0.05, \;\; H_a : p > 0.05.$$

This is Scenario (10); I'll use Table 2.6. Our test stat is (using data from no. 6)

$$z = \frac{(0.14 - 0.05)}{\sqrt{\frac{0.05(1-0.05)}{500}}} = 9.23 > 3.090 = z_{0.001} = z_\alpha,$$

thus we reject H_0; the data suggests, at significance level 0.001, that more than five percent of people are bald.

8. Confidence interval for proportion (in this case p is the proportion of all children who are overexcited) is Scenario (9), Table 2.4. Our data is

$$n = 400, x = 15, \hat{p} = \frac{15}{400} = 0.0375,$$

with $(1-\alpha) = 0.95$, so that $z_{\frac{\alpha}{2}} = 1.96$, giving us

$$0.0375 \pm 1.96\sqrt{\frac{0.0375(1-0.0375)}{400}} \sim 0.0375 \pm 0.0186 = (0.0189, 0.0561).$$

9. Let p be as in no. 8. The claim we are testing is $p \neq 0.12$, so here is our hypothesis test:

$$H_0 : p = 0.12, \;\; H_a : p \neq 0.12.$$

(Single) proportion hypothesis test is Scenario (10), Table 2.5 or 2.6.
From no. 8, $n = 400$ and $\hat{p} = 0.0375$, thus our test stat is

$$z = \frac{(0.0375 - 0.12)}{\sqrt{\frac{(0.12)(0.88)}{400}}} \sim -5.078;$$

this is too large to get a P-value, so I'll use Table 2.6, with $\alpha = 0.05$:

$$|z| = 5.078 \geq 1.96 = z_{0.025} = z_{\frac{\alpha}{2}},$$

thus we reject H_0; at significance level 0.05, there is compelling evidence that the proportion of all children who are overexcited is not 12 percent.

10. This is Scenario (3), Table 2.4, for μ the average sugar content of all BGs:

$$n = 5, \overline{x} = 3, s = 0.6, (1-\alpha) = 0.9,$$

so that, with $(5-1)=4$ degrees of freedom, $t_{\frac{\alpha}{2}} = t_{0.05} = 2.132$, and our confidence interval is

$$3 \pm 2.132\left(\frac{0.6}{\sqrt{5}}\right) \sim 3 \pm 0.572 = (2.428, 3.572).$$

11. Again Scenario (3), Table 2.4. Let's do a 99% confidence interval, so that, with 24 degrees of freedom,

$$\overline{x} \pm t_{0.005}\frac{s}{\sqrt{n}} = 4 \pm 2.797\frac{1.5}{\sqrt{25}} = 4 \pm 0.8391 = (3.1609, 4.8391).$$

12. This is Scenario (1), and, since we're instructed to use a P-value, we'll use Table 2.5. As usual, H_a is our "claim" being tested, so here is our hypothesis test:

$$H_0 : \mu = 75, \quad H_a : \mu > 75.$$

Our data is

$$n = 16, \overline{x} = 80, \sigma = 10,$$

so our test stat is

$$z = \frac{80-75}{\frac{10}{\sqrt{16}}} = 2,$$

so our P-value is

$$P(Z > 2) = 0.0228.$$

Since no significance level α is given, we "should" (social convention) use $\alpha = 0.01$ or 0.05.

At significance level 0.01, we do not reject H_0, since the P-value is greater than 0.01, while at significance level 0.05, we *do* reject H_0, since the P-value is less than or equal to 0.05.

In summary: at significance level 0.01, the data is not sufficient to support the claim, but at significance level 0.05, the data is sufficient to support the claim.

13. In no. 12, $\sigma = 10$ changes to $s = 10$, and Z changes to t, with $(16-1) = 15$ degrees of freedom, thus our test stat is

$$t = \frac{80-75}{\frac{10}{\sqrt{16}}} = 2,$$

and our P-value is now

$$P(T_{15} > 2) = 0.032,$$

and we get the same conclusions as in no. 12.

14. This is Scenario (2), since the sample size $n = 100$, with parameter μ equal to average daily intake of Vitamin Z, testing $\mu \neq 15$; that'll be the hypothesis test

$$H_0 : \mu = 15, \quad H_a : \mu \neq 15,$$

with data

$$n = 100, \overline{x} = 13, s = 8,$$

giving us a test stat of

$$z = \frac{13-15}{\frac{8}{\sqrt{100}}} = -2.5.$$

Once again, no significance level α is given, so we should use $\alpha = 0.01$ or 0.05, which motivates us to use P-values (Table 2.5), because a P-value may be used immediately for any significance level:

$$P\text{-value} = 2P(Z > |-2.5|) = 2(0.0062) = 0.0124.$$

At significance level 0.01, since P-value is greater than 0.01, we do not reject H_0: data does not indicate the intake is significantly different than recommended.

At significance level 0.05, since P-value is less than or equal to 0.05, we reject H_0: data does indicate the intake is significantly different than recommended.

15. The parameter here is p defined to be the proportion of all wolverines that are rabid. For a hypothesis test (Table 2.5 or 2.6), we use Scenario (10), with data

$$n = 400, x = 24, \hat{p} = \frac{24}{400} = 0.06.$$

Since we want evidence for $p < 0.1$, our hypothesis test is

$$H_0 : p = 0.1, \ \ H_a : p < 0.1,$$

with test stat

$$z = \frac{0.06 - 0.1}{\sqrt{\frac{0.1(1-0.1)}{400}}} \sim -2.67,$$

producing a P-value of

$$P(Z < -2.67) = P(Z > 2.67) = 0.0038 \leq 0.05 \equiv \alpha,$$

thus we reject H_0; at significance level 0.05, there is compelling evidence that fewer than ten percent of all wolverines are rabid.

16. Our parameter is again proportion, mainly p defined to be the proportion of Martians that have more than five fingers on each hand. As in no. 15, our hypothesis test is

$$H_0 : p = 0.2, \ \ H_a : p > 0.2,$$

with

$$n = 60, x = 15, \hat{p} = \frac{15}{60} = 0.25,$$

leading to (see Scenario (10)) test stat

$$z = \frac{0.25 - 0.2}{\sqrt{\frac{0.2(1-0.2)}{60}}} \sim 0.97,$$

producing a P-value

$$P(Z > 0.97) = 0.166 > 0.1 = \alpha,$$

so we don't reject H_0; no, we cannot conclude, at significance level 0.1, that more than 20 percent of Martians have more than five fingers on each hand.

17. This problem involves a difference in population means, $(\mu_1 - \mu_2)$, where I am choosing the subscript 1 for Earthling height and 2 for Martian height. Since we are given the standard deviations of both (all) Earthlings and Martians, we are in Scenario (4) of Table 2.3.

Data for Earthlings:

$$n_1 = 10, \overline{x}_1 = \frac{25}{10} = 2.5, \sigma_1 = 2.$$

Data for Martians:

$$n_2 = 7, \overline{x}_2 = \frac{14}{7} = 2, \sigma_2 = 5.$$

(a) From Table 2.4 and Scenario (4), we need $z_{\frac{\alpha}{2}} = z_{0.05} = 1.645$, then our confidence interval is

$$(2.5 - 2) \pm 1.645\sqrt{\frac{2^2}{10} + \frac{5^2}{7}} \sim 0.5 \pm 3.28 = (-2.78, 3.78).$$

(b) "Earthlings taller than Martians" is $(\mu_1 - \mu_2) > 0$, so make that H_a, giving us the following hypothesis test:

$$H_0 : (\mu_1 - \mu_2) = 0 \ \ H_a : (\mu_1 - \mu_2) > 0.$$

Our test stat in Scenario (4) is

$$z = \frac{(2.5 - 2) - 0}{\sqrt{\frac{2^2}{10} + \frac{5^2}{7}}} \sim 0.251,$$

giving us a P-value of

$$P(Z > 0.25) = 0.4013 > 0.1 \equiv \alpha,$$

so we don't reject H_0; data does not suggest, at significance level 0.1, that Earthlings are taller than Martians.

18. Now we are in Scenario (6), with everything as in no. 18, except s_1 rather than σ_1 equals 2, s_2 rather than σ_2 equals 5, and $\sigma_1 = \sigma_2$.

Instead of $z_{0.05}$, we need, with $(10+7-2) = 15$ degrees of freedom, $t_{0.05} = 1.753$. Scenario (6) also requires

$$s_p^2 = \frac{1}{(10+7-2)}\left[(10-1)2^2 + (7-1)5^2\right] = 12.4.$$

(a) $(2.5-2) \pm 1.753\sqrt{12.4}\sqrt{\frac{1}{10}+\frac{1}{7}} \sim 0.5 \pm 3.04 = (-2.54, 3.54)$.

(b) The test stat, with 15 degrees of freedom, is

$$t = \frac{(2.5-2)-0}{\sqrt{12.4}\sqrt{\frac{1}{10}+\frac{1}{7}}} \sim 0.288,$$

so our P-value is approximately

$$P(T_{15} > 0.3) = 0.384,$$

and we reach the same conclusion as in no. 17.

19. Since there is no mention of σ_1 versus σ_2, we must assume $\sigma_1 \neq \sigma_2$, putting us into Scenario (7).

We need ν, the degrees of freedom, from the terrible formula in Scenario (7):

$$\frac{(\frac{3^2}{6}+\frac{5^2}{12})^2}{\left[\frac{1}{(6-1)}(\frac{3^2}{6})^2 + \frac{1}{(12-1)}(\frac{5^2}{12})^2\right]} \sim 15.2,$$

thus we choose $\nu = 15$ degrees of freedom.

(a) $(1-\alpha) = 0.99$ implies that $t_{\frac{\alpha}{2}} = t_{0.005} = 2.947$, so our confidence interval is

$$(8-6) \pm 2.947\sqrt{\frac{3^2}{6}+\frac{5^2}{12}} \sim 2 \pm 5.58 = (-3.58, 7.58).$$

(b) Our test stat, still with 15 degrees of freedom, is

$$t = \frac{(8-6)-(-1)}{\sqrt{\frac{3^2}{6}+\frac{5^2}{12}}} \sim 1.6,$$

causing a P-value of

$$P(T_{15} > 1.6) = 0.065 \leq 0.1 \equiv \alpha,$$

thus we reject H_0; at significance level 0.1, the data suggests that $(\mu_1 - \mu_2) > (-1)$.

20. This is paired data, so we are forced into Scenario (8). First, we need to put the data in terms of $D \equiv (X_1 - X_2)$.

person	1	2	3	4	5	6	7	8	9
$d \equiv (x_1 - x_2)$	7.62	8	9.09	6.06	1.39	16.07	8.4	8.89	2.88

We need $\overline{d}$ and s_d; we will use the Computational Formula 1.3.

$$(d_1 + d_2 + \cdots + d_9) = (7.62 + 8 + 9.09 + \ldots) = 68.4$$

and

$$(d_1^2 + d_2^2 + \cdots + d_9^2) = (7.62^2 + 8^2 + 9.09^2 + \ldots) = 659.48,$$

thus

$$\bar{d} = \frac{68.4}{9} = 7.6 \quad \text{and} \quad s_d^2 = \frac{1}{(9-1)}\left[659.48 - \frac{1}{9}(68.4)^2\right] = 17.455.$$

We are testing

$$H_0 : \mu_d = 5, \;\; H_a : \mu_d > 5,$$

with test stat

$$t = \frac{7.6-5}{\frac{\sqrt{17.455}}{\sqrt{9}}} \sim 1.9,$$

thus, since we have $(9-1) = 8$ degrees of freedom, our P-value is

$$P(T_8 > 1.9) = 0.047.$$

Once again we were not given sufficient information for an unambiguous answer; mainly, we weren't given a signficance level α. The socially correct statistician knows what to do: choose $\alpha = 0.01$ or 0.05.

Since the P-value is greater than 0.01, at significance level 0.01 the claim is not supported by the data.

Since the P-value is less than or equal to 0.05, at significance level 0.05 the claim is supported by the data.

21. This is concerned with the difference of two proportions, $(p_1 - p_2)$; let's have the subscript 1 for Brand X, the subscript 2 for Brand Y.

Here's our data:

$$n_1 = 100, x_1 = 15, \hat{p}_1 = \frac{15}{100} = 0.15, n_2 = 400, x_2 = 24, \hat{p}_2 = \frac{24}{400} = 0.06.$$

(a) For confidence intervals for $(p_1 - p_2)$, we use Scenario (11) in Table 2.4. This means we need

$$s_{\hat{p}_1 - \hat{p}_2} = \sqrt{\frac{0.15(1-0.15)}{100} + \frac{0.06(1-0.06)}{400}} \sim 0.038.$$

Since $(1-\alpha) = 0.95$, $z_{\frac{\alpha}{2}} = z_{0.025} = 1.96$, thus our interval is

$$(0.15 - 0.06) \pm (1.96)(0.038) \sim 0.09 \pm 0.074 = (0.016, 0.164).$$

(b) For hypothesis tests for $(p_1 - p_2)$, we use Scenario (12). A "higher proportion" means $(p_1 - p_2) > 0$, so here is our hypothesis test:

$$H_0 : (p_1 - p_2) = 0, \;\; H_a : (p_1 - p_2) > 0.$$

For our test stat, we first need

$$\hat{p}_3 = \frac{(15+24)}{(100+400)} = 0.078,$$

giving us

$$z = \frac{(0.15 - 0.06) - 0}{\sqrt{0.078(1-0.078)(\frac{1}{100} + \frac{1}{400})}} \sim 3.00,$$

for a P-value

$$P(Z > 3.0) = 0.0013 \leq 0.05 \equiv \alpha,$$

thus we reject H_0; the data implies, at signficance level 0.05, that Brand X leaves a higher proportion of stains than Brand Y.

22. Again a difference of proportions for a hypothesis test; that means Scenario (12). Here's the data, with children getting the subscript 1, adults subscript 2:

$$n_1 = 100, x_1 = 55, \hat{p}_1 = \frac{55}{100} = 0.55, n_2 = 400, x_2 = 210, \hat{p}_2 = \frac{210}{400} = 0.525.$$

"Significant difference" between proportions means $(p_1 - p_2) \neq 0$, so our hypothesis test is

$$H_0 : (p_1 - p_2) = 0, \quad H_a : (p_1 - p_2) \neq 0.$$

We need

$$\hat{p}_3 = \frac{(55 + 210)}{(100 + 400)} = 0.53,$$

so that our test stat is

$$z = \frac{(0.55 - 0.525) - 0}{\sqrt{0.53(1 - 0.53)(\frac{1}{100} + \frac{1}{400})}} \sim 0.45,$$

with a P-value of

$$2P(Z > |0.45|) = 0.6528,$$

much larger than either of the fashionable signficances 0.01 or 0.05, so we don't reject H_0; there is not a significant difference between children and adults, regarding a candy store.

Alternatively, if we used Table 2.6 instead of 2.5, we would compare

$$|z| \sim 0.45 < z_{\frac{\alpha}{2}},$$

for α equal to 0.01 or 0.05, implying that we don't reject H_0.

23. We're testing a statement about a single average, so our parameter is μ, as in Scenarios (1)–(3). Normality is assumed, with sample size $n = 5 \leq 40$, so we are in Scenario (3), with $(5 - 1) = 4$ degrees of freedom.

With $\mu \equiv$ average number of calories of SS, our hypothesis test is

$$H_0 : \mu = 14, \quad H_a : \mu < 14.$$

We'll use Computational Formula 1.3 to get $\overline{x}$ and s:

$$(12 + 11 + 9 + 12 + 16) = 60, \quad (12^2 + 11^2 + 9^2 + 12^2 + 16^2) = 746,$$

so

$$\overline{x} = \frac{60}{5} = 12, \quad s^2 = \frac{1}{(5-1)}\left[746 - \frac{1}{5}(60)^2\right] = 6.5.$$

Our test stat, with $(5 - 1) = 4$ degrees of freedom, is

$$t = \frac{12 - 14}{\sqrt{\frac{6.5}{5}}} \sim -1.75,$$

producing a P-value of

$$P(T_4 < -1.75) = P(T_4 > 1.75) \sim P(T_4 > 1.8) = 0.073 \leq 0.1,$$

thus we reject H_0; at significance level 0.1, the claim is supported by the data.

24. We are testing a difference between averages, which we denote as $(\mu_1 - \mu_2)$; let's have subscript 1 for Tasmanian weights and subscript 2 for mainland Australian weights. The data given is

$$n_1 = 10, \overline{x}_1 = 80, s_1 = 4, n_2 = 20, \overline{x}_2 = 75, s_2 = 5, \sigma_1 = \sigma_2.$$

That puts us in Scenario (6), which means we need

$$s_p^2 = \frac{1}{10 + 20 - 2}\left[(10-1)4^2 + (20-1)5^2\right] \sim 22.1.$$

We seek a 99% confidence interval, thus $(1 - \alpha) = 0.99$ implies that $\frac{\alpha}{2} = 0.005$. Our critical value has $(10 + 20 - 2) = 28$ degrees of freedom, thus our critical value is

$$t_{0.005} = 2.763.$$

We appear to have the pieces for Table 2.4, Scenario (6):

$$(80 - 75) \pm 2.763\sqrt{22.1}\sqrt{\frac{1}{10} + \frac{1}{20}} \sim 5 \pm 5.03 = (-0.03, 10.03).$$

25. This is comparing proportions; equivalently, looking at $(p_1 - p_2)$, where p_1 is proportion of fish eaters who solve a puzzle, p_2 likewise for non-fish eaters. "Fish makes you smarter" translates as $(p_1 - p_2) > 0$, so we have the hypothesis test

$$H_0 : (p_1 - p_2) = 0, \quad H_a : (p_1 - p_2) > 0.$$

Since it's a hypothesis test for a difference of proportions, we are in Scenario (12).

Here is the data:

$$n_1 = 25, x_1 = 15, \hat{p}_1 = \frac{15}{25} = 0.6, n_2 = 20, x_2 = 10, \hat{p}_2 = \frac{10}{20} = 0.5.$$

We need

$$\hat{p}_3 = \frac{15 + 10}{25 + 20} = 0.556,$$

so our test stat is

$$z = \frac{(0.6 - 0.5) - 0}{\sqrt{0.556(1 - 0.556)(\frac{1}{25} + \frac{1}{20})}} \sim 0.67 > 0.05 \equiv \alpha,$$

so we don't reject H_0: at significance level 0.05, the data does not suggest that fish makes you smarter.

26. This is comparing means, that is, doing inference on $(\mu_1 - \mu_2)$. Let's have subscript 1 for Neptune and subscript 2 for Pluto. Here's the data:

$$n_1 = 100, \overline{x}_1 = 6.2, s_1 = 2, n_2 = 200, \overline{x}_2 = 5.7, s_2 = 3.$$

The large values of n_1 and n_2 put us, among all the possible $(\mu_1 - \mu_2)$ scenarios, in Scenario (5).

(a) This is

$$H_0 : (\mu_1 - \mu_2) = 0, \quad H_a : (\mu_1 - \mu_2) \neq 0.$$

Our test stat is

$$z = \frac{(6.2 - 5.7) - 0}{\sqrt{\frac{2^2}{100} + \frac{3^2}{200}}} \sim 1.71,$$

giving us a P-value of

$$2P(Z > |1.71|) = 2(0.0436) = 0.0872 > 0.01 \equiv \alpha,$$

so we do not reject H_0; the data does not suggest that the average lengths are different, at significance level 0.01.

(b) Since $(1 - \alpha) = 0.99$, we need $z_{0.005} = 2.576$; now we follow Scenario (5), Table 2.4:

$$(6.2 - 5.7) \pm 2.576\sqrt{\frac{2^2}{100} + \frac{3^2}{200}} \sim 0.5 \pm 0.751 = (-0.251, 1.251).$$

27. Here's another comparison of means, that is, inference on $(\mu_1 - \mu_2)$, where we use the subscript 1 for fish eaters and the subscript 2 for non-fish eaters.

To use our data, we need to equate higher scores with being smart. Thus our hypothesis test is

$$H_0 : (\mu_1 - \mu_2) = 0, \quad (\mu_1 - \mu_2) > 0.$$

To choose among Scenarios (4)–(8), let's write down the data given:

$$n_1 = 8, \overline{x}_1 = 65, \sigma_1 = 3, n_2 = 10, \overline{x}_2 = 60, \sigma_2 = 5.$$

Since σ_1 and σ_2 are given, we use Scenario (4). That means our test stat is

$$z = \frac{(65 - 60) - 0}{\sqrt{\frac{3^2}{8} + \frac{5^2}{10}}} \sim 2.63,$$

so that our P-value is

$$P(Z > 2.63) = 0.0043 \leq 0.01 \equiv \alpha,$$

thus we reject H_0: the data suggests, at significance level 0.01, that fish makes you smarter (on average).

28. This is proportion; specifically, the proportion, call it p, of wolverines who are rabid. Our data is

$$n = 400, x = 28, \hat{p} = \frac{28}{400} = 0.07.$$

(a) For an upper confidence bound for p, we use Scenario (9), Table 2.4. Since $(1-\alpha) = 0.95$, $z_\alpha = z_{0.05} = 1.645$, so our 95% upper confidence bound is

$$0.07 + 1.645\sqrt{\frac{0.07(1-0.07)}{400}} \sim 0.07 + 0.021 = 0.091.$$

(b) For the hypothesis test

$$H_0 : p = 0.1, \quad H_a : p < 0.1,$$

use Scenario (10), giving us the test stat

$$z = \frac{0.07 - 0.1}{\sqrt{\frac{0.1(1-0.1)}{400}}} = -2,$$

so that the P-value is

$$P(Z < -2) = P(Z > 2) = 0.0228 \leq 0.05 \equiv \alpha,$$

making us reject H_0; the data suggests that fewer than ten percent of wolverines are rabid.

29. This involves the proportion, call it p, of college students who like onions on their hamburger. Since it's a hypothesis test

$$H_0 : p = 0.3, \quad H_a : p > 0.3,$$

we use Scenario (10). Here's our data:

$$n = 100, x = 42, \hat{p} = \frac{42}{100} = 0.42.$$

Our test stat is

$$z = \frac{0.42 - 0.3}{\sqrt{\frac{0.3(1-0.3)}{100}}} \sim 2.62$$

thus our P-value is

$$P(Z > 2.62) = 0.0044 \leq 0.01 \leq 0.05,$$

thus (since no significance level given, use 0.05 or 0.01 for α) we reject H_0; at significance level 0.01 or 0.05, the data provides strong evidence that the proportion of all college students who like onions on their hamburgers exceeds the proportion for all people.

30. This is inference on a single population average μ (defined to be the average cholesterol level of all wolverines), so we must choose from Scenarios (1)–(3). Since no population standard deviation is given, and the sample size is less than or equal to 40, we choose Scenario (3).

Here's the data:

$$n = 6, \overline{x} = 90, s = 12,$$

and $(1-\alpha) = 0.95$ implies that $\frac{\alpha}{2} = 0.025$, so that $t_{\frac{\alpha}{2}} = 2.571$, with $(6-1) = 5$ degrees of freedom, and, from Table 2.4, our confidence interval is

$$90 \pm 2.571\frac{12}{\sqrt{6}} \sim 90 \pm 12.6 = (77.4, 102.6).$$

31. Let μ be the mean test score of all Martians. Our hypothesis test is

$$H_0 : \mu = 100, \quad H_a : \mu \neq 100.$$

We are given normality and population standard deviation $\sigma = 15$, so we are in Scenario (1). For our test stat, we also need

$$\overline{x} = \frac{1}{5}(114 + 100 + 104 + 129 + 153) = 120.$$

Our test stat is

$$z = \frac{120 - 100}{\frac{15}{\sqrt{5}}} \sim 2.98,$$

giving us a P-value of

$$2P(Z > |2.98|) = 0.0028 \leq 0.01 \leq 0.05,$$

so we reject H_0; at significance level 0.01 or 0.05 (chosen because no significance level was given), there is evidence that Martian test scores, on average, differ from 100.

32. This is Scenario (1) again, since we are given normality and $\sigma = 7$.

Let's calculate $\overline{x} = \frac{1}{9}(31 + 31 + 43 + 40 + 20 + 35 + +30 + 30 + 10) = 30$. $(1 - \alpha) = 0.99$ implies that $z_{\frac{\alpha}{2}} = z_{0.005} = 2.576$, so our interval is

$$30 \pm 2.576\frac{7}{\sqrt{9}} \sim 30 \pm 6.01 = (23.99, 36.01).$$

33. Letting μ be the average length of all scumslugs, we are given normality and can calculate $\overline{x}$ and s for the data given, thus we are in Scenario (3).

We'll use the Computational Formula 1.3:

$$(56 + 68 + 52 + 24 + 50) = 250 \text{ and } (56^2 + 68^2 + 52^2 + 24^2 + 50^2) = 13,540,$$

thus

$$\overline{x} = \frac{1}{5}(250) = 50 \text{ and } s^2 = \frac{1}{(5-1)}\left[13,540 - \frac{1}{5}(250)^2\right] = 260.$$

Also $(1 - \alpha) = 0.99$ implies that $\frac{\alpha}{2} = 0.005$. Staring at Table 2.4, Scenario (3), tells us to get, with $(5 - 1) = 4$ degrees of freedom,

$$t_{0.005} = 4.604,$$

so here's our interval:

$$50 \pm 4.604(\frac{\sqrt{260}}{\sqrt{5}}) \sim 50 \pm 33.20 = (16.80, 83.20).$$

34. Assuming a positive connection between "smart" and "score on math exam" (we have to work with whatever data is given), our parameter should be μ defined to be the average math exam score of all fish eaters.

Here is the hypothesis test for the claim "fish makes you smarter":

$$H_0 : \mu = 70, \ H_a : \mu > 70.$$

Here're the numbers:

$$n = 16, \overline{x} = 85, s = 3.$$

We are in Scenario (3), so our test stat is

$$t = \frac{85 - 70}{\frac{3}{\sqrt{16}}} = 20.$$

P-value is difficult here, so let's use Table 2.6, Scenario (3). For $(16 - 1) = 15$ degrees of freedom, our relevant critical value is

$$t_\alpha = t_{0.01} = 2.602,$$

thus, since

$$t = 20 > 2.602 = t_{0.01},$$

we reject H_0; at significance level 0.01, the data suggests that fish makes you smarter.

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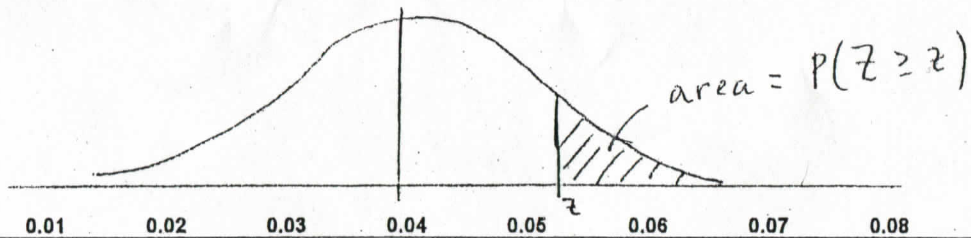
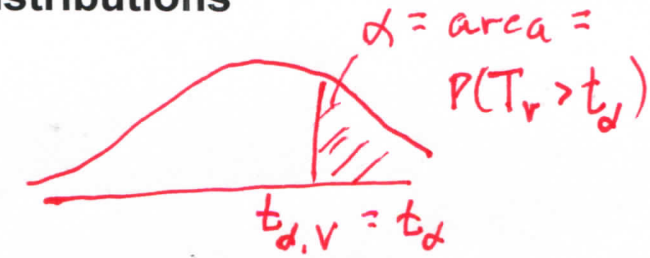
The Standard Normal Distribution (Areas in the Right Tail)



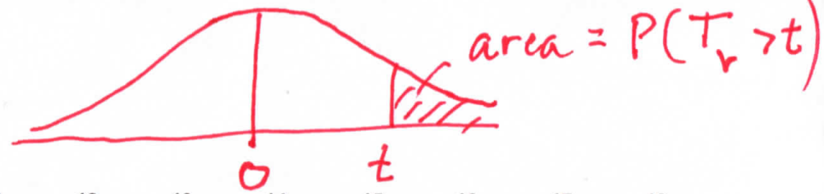
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010

Critical Values for t Distributions



v \ α	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
1	3.078	6.314	12.706	31.821	63.657	318.310	636.620
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.767
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
32	1.309	1.694	2.037	2.449	2.738	3.365	3.622
34	1.307	1.691	2.032	2.441	2.728	3.348	3.601
36	1.306	1.688	2.028	2.434	2.719	3.333	3.582
38	1.304	1.686	2.024	2.429	2.712	3.319	3.566
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
50	1.299	1.676	2.009	2.403	2.678	3.262	3.496
60	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.090	3.291

t Curve Tail Areas



t	v = 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
0.0	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
0.1	0.468	0.465	0.463	0.463	0.462	0.462	0.462	0.461	0.461	0.461	0.461	0.461	0.461	0.461	0.461	0.461	0.461	0.461
0.2	0.437	0.430	0.427	0.426	0.425	0.424	0.424	0.423	0.423	0.423	0.423	0.422	0.422	0.422	0.422	0.422	0.422	0.422
0.3	0.407	0.396	0.392	0.390	0.388	0.387	0.386	0.386	0.386	0.385	0.385	0.385	0.384	0.384	0.384	0.384	0.384	0.384
0.4	0.379	0.364	0.358	0.355	0.353	0.352	0.351	0.350	0.349	0.349	0.348	0.348	0.348	0.347	0.347	0.347	0.347	0.347
0.5	0.352	0.333	0.326	0.322	0.319	0.317	0.316	0.315	0.315	0.314	0.313	0.313	0.313	0.312	0.312	0.312	0.312	0.312
0.6	0.328	0.305	0.295	0.290	0.287	0.285	0.284	0.283	0.282	0.281	0.280	0.280	0.279	0.279	0.279	0.278	0.278	0.278
0.7	0.306	0.278	0.267	0.261	0.258	0.255	0.253	0.252	0.251	0.250	0.249	0.249	0.248	0.247	0.247	0.247	0.247	0.246
0.8	0.285	0.254	0.241	0.234	0.230	0.227	0.225	0.223	0.222	0.221	0.220	0.220	0.219	0.218	0.218	0.218	0.217	0.217
0.9	0.267	0.232	0.217	0.210	0.205	0.201	0.199	0.197	0.196	0.195	0.194	0.193	0.192	0.191	0.191	0.191	0.190	0.190
1.0	0.250	0.211	0.196	0.187	0.182	0.178	0.175	0.173	0.172	0.170	0.169	0.169	0.168	0.167	0.167	0.166	0.166	0.165
1.1	0.235	0.193	0.176	0.167	0.162	0.157	0.154	0.152	0.150	0.149	0.147	0.146	0.146	0.144	0.144	0.144	0.143	0.143
1.2	0.221	0.177	0.158	0.148	0.142	0.138	0.135	0.132	0.130	0.129	0.128	0.127	0.126	0.124	0.124	0.124	0.123	0.123
1.3	0.209	0.162	0.142	0.132	0.125	0.121	0.117	0.115	0.113	0.111	0.110	0.109	0.108	0.107	0.107	0.106	0.105	0.105
1.4	0.197	0.148	0.128	0.117	0.110	0.106	0.102	0.100	0.098	0.096	0.095	0.093	0.092	0.091	0.091	0.090	0.090	0.089
1.5	0.187	0.136	0.115	0.104	0.097	0.092	0.089	0.086	0.084	0.082	0.081	0.080	0.079	0.077	0.077	0.077	0.076	0.075
1.6	0.178	0.125	0.104	0.092	0.085	0.080	0.077	0.074	0.072	0.070	0.069	0.068	0.067	0.065	0.065	0.065	0.064	0.064
1.7	0.169	0.116	0.094	0.082	0.075	0.070	0.065	0.064	0.062	0.060	0.059	0.057	0.056	0.055	0.055	0.054	0.054	0.053
1.8	0.161	0.107	0.085	0.073	0.066	0.061	0.057	0.055	0.053	0.051	0.050	0.049	0.048	0.046	0.046	0.045	0.045	0.044
1.9	0.154	0.099	0.077	0.065	0.058	0.053	0.050	0.047	0.045	0.043	0.042	0.041	0.040	0.038	0.038	0.038	0.037	0.037
2.0	0.148	0.092	0.070	0.058	0.051	0.046	0.043	0.040	0.038	0.037	0.035	0.034	0.033	0.032	0.032	0.031	0.031	0.030
2.1	0.141	0.085	0.063	0.052	0.045	0.040	0.037	0.034	0.033	0.031	0.030	0.029	0.028	0.027	0.027	0.026	0.025	0.025
2.2	0.136	0.079	0.058	0.046	0.040	0.035	0.032	0.029	0.028	0.026	0.025	0.024	0.023	0.022	0.022	0.021	0.021	0.021
2.3	0.131	0.074	0.052	0.041	0.035	0.031	0.027	0.025	0.023	0.022	0.021	0.020	0.019	0.018	0.018	0.018	0.017	0.017
2.4	0.126	0.069	0.048	0.037	0.031	0.027	0.024	0.022	0.020	0.019	0.018	0.017	0.016	0.015	0.015	0.014	0.014	0.014
2.5	0.121	0.065	0.044	0.033	0.027	0.023	0.020	0.018	0.017	0.016	0.015	0.014	0.013	0.012	0.012	0.012	0.011	0.011
2.6	0.117	0.061	0.040	0.030	0.024	0.020	0.018	0.016	0.014	0.013	0.012	0.012	0.011	0.010	0.010	0.010	0.009	0.009
2.7	0.113	0.057	0.037	0.027	0.021	0.018	0.015	0.014	0.012	0.011	0.010	0.010	0.009	0.008	0.008	0.008	0.008	0.007
2.8	0.109	0.054	0.034	0.024	0.019	0.016	0.013	0.012	0.010	0.009	0.009	0.008	0.008	0.007	0.007	0.006	0.006	0.006
2.9	0.106	0.051	0.031	0.022	0.017	0.014	0.011	0.010	0.009	0.008	0.007	0.007	0.006	0.005	0.005	0.005	0.005	0.005
3.0	0.102	0.048	0.029	0.020	0.015	0.012	0.010	0.009	0.007	0.007	0.006	0.006	0.005	0.004	0.004	0.004	0.004	0.004
3.1	0.099	0.045	0.027	0.018	0.013	0.011	0.009	0.007	0.006	0.006	0.005	0.005	0.004	0.004	0.004	0.003	0.003	0.003
3.2	0.096	0.043	0.025	0.016	0.012	0.009	0.008	0.006	0.005	0.005	0.004	0.004	0.003	0.003	0.003	0.003	0.003	0.002
3.3	0.094	0.040	0.023	0.015	0.011	0.008	0.007	0.005	0.005	0.004	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002
3.4	0.091	0.038	0.021	0.014	0.010	0.007	0.006	0.005	0.004	0.003	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002
3.5	0.089	0.036	0.020	0.012	0.009	0.006	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.001	0.001	0.001
3.6	0.086	0.035	0.018	0.011	0.008	0.006	0.004	0.004	0.003	0.002	0.002	0.002	0.002	0.001	0.001	0.001	0.001	0.001
3.7	0.084	0.033	0.017	0.010	0.007	0.005	0.004	0.003	0.002	0.002	0.002	0.002	0.001	0.001	0.001	0.001	0.001	0.001
3.8	0.082	0.031	0.016	0.010	0.006	0.004	0.003	0.003	0.002	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
3.9	0.080	0.030	0.015	0.009	0.006	0.004	0.003	0.002	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
4.0	0.078	0.029	0.014	0.008	0.005	0.004	0.003	0.002	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.000	0.000